Advanced Placement

Physics

Study Guide

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Important Note to the Reader: Many of the review sections in Newtonian Mechanics and Electricity and Magnetism are written for both the Advanced Placement B (life science majors) and C (physical science majors) courses. In these reviews the additional Advanced Placement C course information will be boxed within a border that appears like the border surrounding this message. This information is required for AP C students. Some sections below are targeted only to AP C students. Instead AP Physics B students will do work in Fluids/Thermal Physics, Waves and Optics, and Modern Physics. These sections do not apply to AP Physics C students

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Graphing

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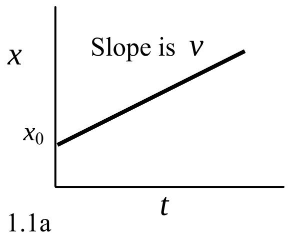
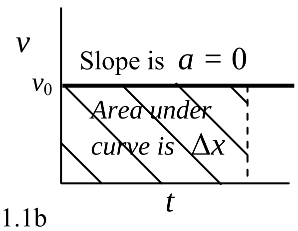
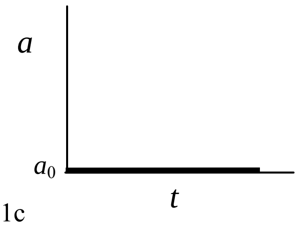
<tr><td>Constant Velocity</td><td>
Fig 1.1a</td><td>
Fig 1.1b</td><td>
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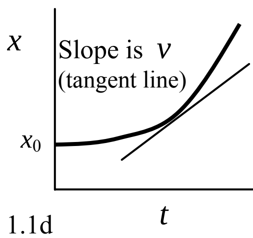
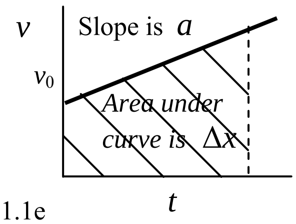
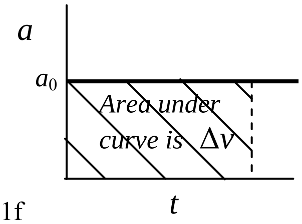
<tr><td>Uniform Acceleration</td><td>
Fig 1.1d</td><td>
Fig 1.1e</td><td>
Fig 1.1f</td></tr>
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Slopes of Curves are an important analytical tool used in physics. Any equation that can be manipulated into the format $y = mx + b$ can be represented and analyzed graphically. As an example: $v = v_0 + at$ can be rearranged slightly into $v = at + v_0$. Compare this equation to the equation of a line. It is apparent that a is the slope and that v_0 is the y-intercept (Fig 1.1b and Fig 1.1e). What equation generates velocity in Fig. 1.1a and Fig 1.1d?

Area Under a Curve is another important graphical tool. Multiply the y-axis (height) by the x-axis (base) and determine if this matches any known equations. For example: Figures 1.1b and 1.1e are velocity-time plots. Simply multiply $v \times t$. This is a rearranged form of the equation $v = x/t$. The form obtained from the graph is $x = vt$, which means that displacement is the area under the velocity-time plot.

Calculus: ***Required in AP Physics C*** (optional for AP Physics B students)***. Calculus is taught in math class. These review sheets will focus on the Physics aspect of solutions. Calculus steps may not be shown. Solution will be up to the student.***

The equations outlined in the previous page work well in the following situations.

- Linear Functions: involving constant velocity and acceleration, as diagrammed in the above graphs (except Fig 1.1d).
- Nonlinear Functions: scenarios where the problem is seeking information about a change in a quantity, Δx or Δv.
- Nonlinear Functions: scenarios where the problem is seeking an average velocity in an interval.

Calculus is needed to find the slopes of nonlinear functions and the areas under nonlinear curves.

1. **Velocity: Slope of the displacement-time curve.**

$$v = \frac{dx}{dt} \qquad \text{example:} \qquad v = \frac{d}{dt}\left(x_0 + v_0 t + \frac{1}{2}at^2\right) = v_0 + at \qquad v = v_0 + at$$

2. **Acceleration: Slope of the velocity-time curve.**

$$a = \frac{dv}{dt} \qquad \text{example:} \qquad a = \frac{d}{dt}\left(v_0 + at\right) = a$$

3. **Velocity: Area under acceleration-time curve.** (Note: if $c = v_0$ cannot be found, then you can only solve for Δv)

$$v = \int a \; dt \qquad \text{example:} \qquad v = \int (a) \; dt = c + at \qquad v = v_0 + at$$

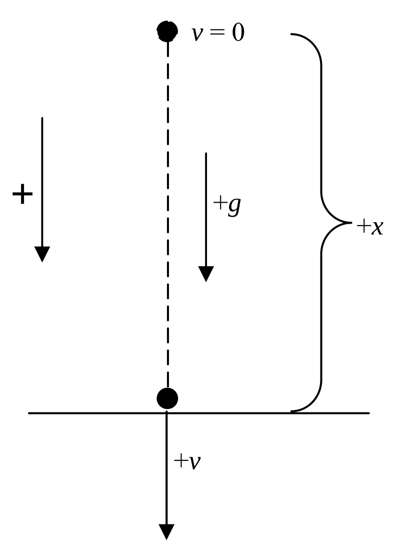
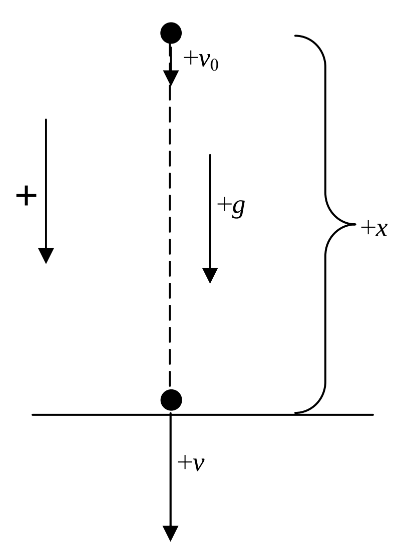
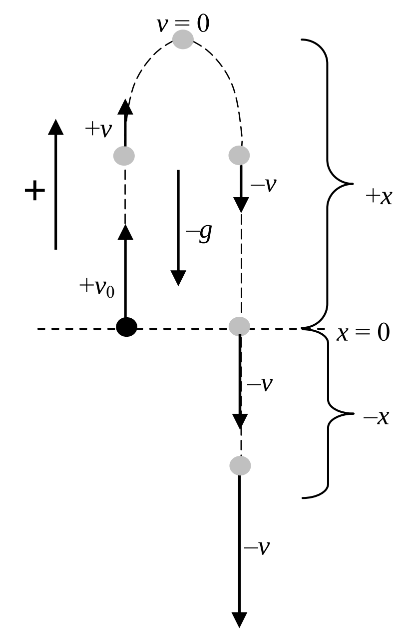
4. **Displacement: Area under the velocity-time curve.** (Note: if $c = x_0$ cannot be found, then you can only solve Δx)

$$x = \int v \; dt \qquad \text{example:} \qquad x = \int (v_0 + at) \; dt = c + v_0 t + \frac{1}{2}at^2 \qquad x = x_0 + v_0 t + \frac{1}{2}at^2$$

Falling Bodies: Objects moving vertically under the influence of gravity. Earth's surface gravity is $g = –9.8$ m/s^2. This speeds objects up (+ acceleration), but is directed downward (–). Objects can be thrown up, down, or just be dropped. They can land below, at the same height, or above the origin. They come to an instantaneous stop at the highest point in flight.

$$\boxed{v_y = v_{0y} + gt} \qquad \boxed{y = y_0 + v_{0y}t + \frac{1}{2}gt^2} \qquad \boxed{v_y{}^2 = v_{0y}{}^2 + 2g\left(y - y_0\right)}$$

The positive and negative signs can cause trouble in these problems. The easiest way to handle the signs is set the direction of initial motion as positive and then to ensure all signs are consistent with this decision. This has one huge benefit. It eliminated the double sign on acceleration. When initial velocity v_0 is used to anchor direction, then a positive acceleration means speeding up and negative acceleration involves slowing down.

Dropped from rest	**Thrown downward**	**Thrown upward**
$v_0 = 0$, but it will move down initially	v_0 is directed downward	v_0 is directed upward
Set downward as positive direction	Set downward as positive direction	Set upward as positive direction
		
Everything is positive and easy	Everything is positive and easy	This is difficult, and depends on where in the flight the problem ends.

Kinematics Problems Involving Changes in the Magnitude of Acceleration

If the magnitude of acceleration changes while solving a kinematics problem then the problem must be solved in separate parts. Unlike displacement x and velocity v, the Kinematic equations do not contain the variables a_0 and a.

Example 1-3: More than one acceleration

A car initially at rest accelerates at 4 m/s^2 while covering a distance of 100 m. Then the car continues at constant velocity for 500 m. Finally it slows to a stop with a deceleration of 3 m/s^2. Determine the total time of this displacement.

Acceleration Phase: $x = \frac{1}{2}at^2$ $\quad t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2(100)}{(4)}} = 7.07\,s$ and $v = v_0 + at = (0) + (4)(7.07) = 28.3\,m/s$

Constant Velocity Phase: $v = \frac{\Delta x}{t}$ $\quad t = \frac{\Delta x}{v} = \frac{(500)}{(28.3)} = 17.7\,s$ $\quad t = \frac{500}{v}$

Deceleration Phase: $v = v_0 + at$ $\quad t = \frac{v - v_0}{a} = \frac{(0) - (28.3)}{(-3)} = 9.43\,s$

Total Time = 7.07 + 17.7 + 9.43 = 34.2 s

1–2 Vectors and Vectors in Kinematics

Scalar: A physical quantity described by a single number and units. A quantity describing ***magnitude*** only.

Vector: Many of the variables in physics equations are vector quantities. Vectors have ***magnitude*** and ***direction***.

Magnitude: Size or extend. The numerical value.

Direction: Alignment or orientation with respect to set location and system of orientation, such as a coordinate axis.

Notation: $\vec{A}$ or $\xrightarrow{A}$ The length of the arrow represents, and is proportional to, the vectors magnitude. The direction the arrow points indicates the direction of the vector.

Negative Vectors: Have the same magnitude as their positive counterpart, but point in the opposite direction.

If $\xrightarrow{A}$ then $\xleftarrow{-A}$

Vector Addition and subtraction

Think of it as vector addition only. The result of adding vectors is called the **<u>result</u>**ant $\vec{R}$.

$\vec{A}+\vec{B}=\vec{R}$ $\qquad \xrightarrow{A} \quad + \quad \xrightarrow{B} \quad = \quad \xrightarrow{R}$

When you need to subtract one vector from another think of the one being subtracted as being a negative vector.

$\vec{A}-\vec{B}$ is really $\vec{A}+\left(-\vec{B}\right)=\vec{R}$ $\qquad \xrightarrow{A} \quad + \quad \xleftarrow{(-B)} \quad = \quad \xrightarrow{R}$

A negative vector has the same length as its positive counterpart, but its direction is reversed. ***<u>This is very important</u>***. In physics a negative number does not always mean a smaller number. Mathematically –2 is smaller than +2, but in physics these numbers have the same magnitude (size), they just point in different directions (180^o apart).

Vector Addition: Parallelogram

A* + *B

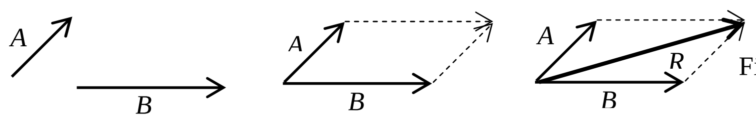


Fig 2.1a

Tip to Tail

A* + *B

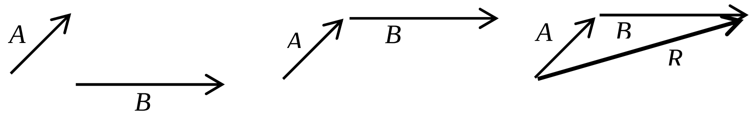


Fig 2.1b

Both methods arrive at the exact same solution since either method is essentially a parallelogram. In some problems one method is advantageous, while in other problems the alternative method is superior.

Reporting Magnitude and Direction:

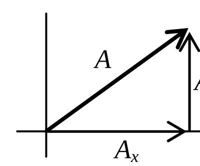


Fig. 2.2a

Component Method: A vector a can be reported by giving the components along the *x*- or *y*-axis. Reporting a vector this way is formally done by employing the unit vectors **i** and **j**. As an example: vector A in fig 2.2a would be $\boxed{A=A_x\mathbf{i}+A_y\mathbf{j}}$, where, $|\mathbf{i}|+|\mathbf{j}|=\mathbf{1}$. There is at third component vector **k** used for three dimensional problems involving the *z*-axis. The vector in Fig. 2.2b shows a numerical application of the component method. In this example you are given the polar coordinates $A=5\ at\ 37^o$. Using trigonometry the components can be established.

$\boxed{A_x=A\cos\theta}=5\cos 37^o=4$ and $\boxed{A_y=A\sin\theta}=5\sin 37^o=3$. Then A can be expressed as follows: $A=4\mathbf{i}+3\mathbf{j}$

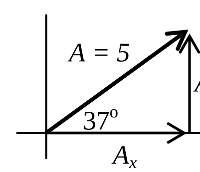


Fig. 2.2b

Polar Coordinates: Vector A in Fig. 2.2b is reported in polar coordinates, $A=5\ at\ 37^o$. This is simply the length of a vector and its angle measured counterclockwise with respect to the positive *x*-axis. (Negative angles are allowed and indicate that direction was measured clockwise from the +*x*-axis. If the component vectors are given, $A=4\mathbf{i}+3\mathbf{j}$, Pythagorean theorem is used to establish the length of the parent vector $\boxed{A=\sqrt{A_x^{\ 2}+A_y^{\ 2}}}=\sqrt{4^2+3^2}=5$.

Arctangent is used to find the direction $\boxed{\theta=\tan^{-1}\frac{A_y}{A_x}}=\tan^{-1}\frac{3}{4}=37^o$. ***But, watch out! The angle arrived at by the arctangent formula may not be the final answer. The quadrant that the final vector lies in must be established, and an adjustment to the angle may be needed in order to provide an answer that extends from the +x-axis.***

Component Advantage In Vector Addition:

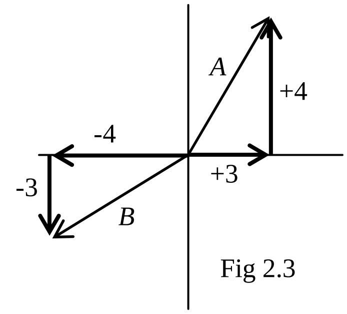


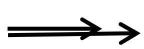
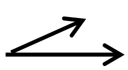
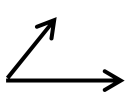
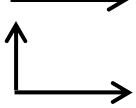
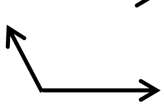
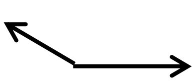
Fig 2.3

1. If vector components along an axis are used, direction can be specified with + and – symbols. Vectors A and B in Fig 2.3 have been converted into components.
   $A = (+3)\,\mathbf{i} + (+4)\,\mathbf{j}$ $\qquad$ $B = (-4)\,\mathbf{i} + (-3)\,\mathbf{j}$
2. This becomes advantageous if vectors A and B need to be added.
   Find the resultant of the x vectors: $\boxed{R_x = A_x + B_x} = (+3) + (-4) = -1$
   Find the resultant of the y vectors: $\boxed{R_y = A_y + B_y} = (+4) + (-3) = +1$
   Then combine them to find R: $\boxed{R = \sqrt{R_x^{\,2} + R_y^{\,2}}} = \sqrt{(-1)^2 + (+1)^2} = 1.41$
   Find the direction: $\boxed{\theta = \tan^{-1}\dfrac{R_y}{R_x}} = \tan^{-1}\dfrac{(+1)}{(-1)} = -45^o$, but this a 2nd quadrant angle and must be adjusted: $180^o - 45^o = 135^o$. The final vector is has a magnitude of 1.41 and a direction of 135°.
3. Finding the components simplify problems throughout physics. In Newtonian Mechanics motion, force, and momentum often act at an angle to the *x*- or *y*-axis. Fortunately these vector quantities can be resolved into component vectors along the *x*- or *y*-axis. In addition, equations for motion, force, and momentum can be calculated in the *x*-direction independent of what is happening in the *y*-direction, and vice versa. The first example of this will be projectile motion. In projectile motion there will be a variety of initial and final of velocities at angles.

Scalar (Dot) Product of Two Vectors: The dot product (**A·B**) of two vectors **A** and **B** is a scalar and is equal to $AB\cos\theta$. This quantity shows how two vectors interact depending on how close to parallel the two vectors are. The magnitude of this scalar is largest when $\theta = 0^o$ (parallel) and when $\theta = 180^o$ (anti-parallel). The scalar is zero when $\theta = 90^o$ (perpendicular).

Vector (Cross) Product: The cross product (**A×B**) of two vectors **A** and **B** is a third vector **C**. The magnitude of this vector is $C = AB\sin\theta$. The direction of this vector is determined by the right-hand-rule. The order that the vectors are multiplied is important (not commutative) If you change the order of multiplication you must change the sign, **A×B = – A×B**. The magnitude of this vector is largest when $\theta = 90^o$ (perpendicular). The vector is zero when $\theta = 0^o$ (parallel) or when $\theta = 180^o$ (anti-parallel).

Examples: In the following examples vector **A** = 3 and **B** = 2. The direction of vector **A** will vary.

Vectors A & B	Scalar (Dot) Product	Vector (Cross Product)
	$(3)(2)\cos 0^o = \boxed{6}$	$(3)(2)\sin 0^o = \boxed{0}$
	$(3)(2)\cos 30^o = \boxed{5.2}$	$(3)(2)\sin 30^o = \boxed{3.0}$
	$(3)(2)\cos 60^o = \boxed{3}$	$(3)(2)\sin 60^o = \boxed{5.2}$
	$(3)(2)\cos 90^o = \boxed{0}$	$(3)(2)\sin 90^o = \boxed{6}$
	$(3)(2)\cos 120^o = \boxed{-3}$	$(3)(2)\sin 120^o = \boxed{5.2}$
	$(3)(2)\cos 150^o = \boxed{-5.2}$	$(3)(2)\sin 150^o = \boxed{3}$
	$(3)(2)\cos 180^o = \boxed{-6}$	$(3)(2)\sin 180^o = \boxed{0}$

Another way: When cos (parallel) appears in a formula you need two vectors that are parallel (+) or anti-parallel (–). Just use your trig skills to find a component of one of the vectors that points in the same direction as the other vector. In the examples above, find the component of vector **B** that is parallel to vector **A**. When sin (perpendicular) appears in a formula, find the component of vector **B** that is perpendicular to vector **A**.

1–3 Motion in Two Dimensions

Relative Velocity: Motion in two dimensions, both at constant velocity. A common example is that of a boat crossing a river. In figure 3.1, a boat leaves perpendicular to the shore with a velocity of v_B. The rivers current, v_R, carries it downstream a distance y. In figure 3.2 the boat aims at an angle of θ upstream in order to end up straight across. There is a triangle formed by the solid velocity vectors, and another formed by the dashed displacement vectors. These triangles are similar triangles. The resultant velocity of the boat will be a combination of the boats own velocity and current. This vector is labeled Σv. This is how fast the boat will appear to be moving as seen from a stationary observation point on shore. When calculating the time to cross the stream use the velocity vector and displacement vector that point in the same direction. In figure 3.1, $v_B = \dfrac{x}{t}$. In figure 3.2, $\sum v = \dfrac{x}{t}$

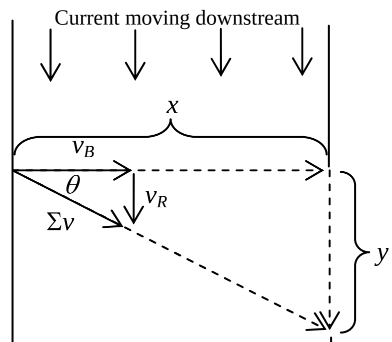


Fig 3.1

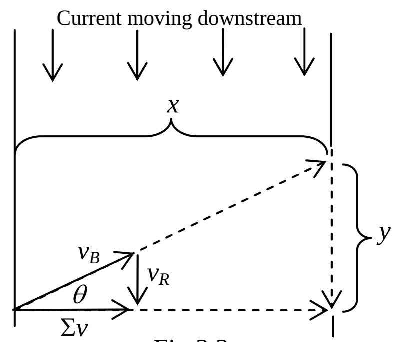


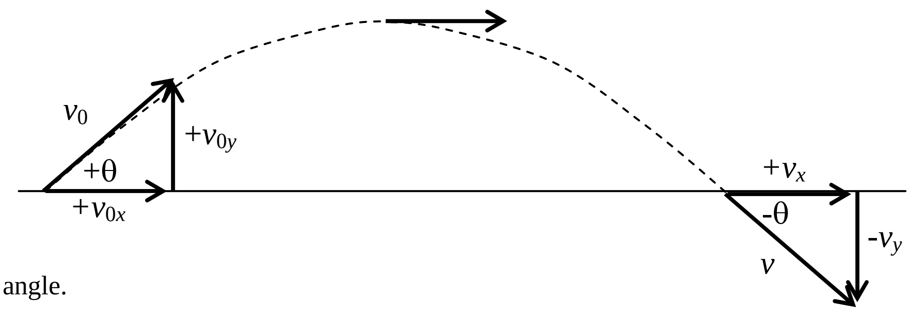
Fig 3.2

Projectile Motion: Motion in one dimension involves acceleration, while the other is at constant velocity.

- In the *x*-direction the velocity is constant, with no acceleration occurring in this dimension.
- In the *y*-direction the acceleration of gravity slows upward motion and enhances downward motion.

Vector Components in Projectile Motion: **The *x*-direction and the *y*-direction are independent of each other.**

Fig 3.3



Now v_0 is at an angle.

Solve for v_{0x}: $\boxed{v_{0x} = v_0 \cos\theta}$ then use v_{0x} in the kinematic equations to solve for v_x.

Solve for v_{0y}: $\boxed{v_{0y} = v_0 \sin\theta}$ then use v_{0y} in the falling body equations to solve for v_y.

v_x and v_y are component vectors. To find v, use Pythagorean Theorem $\boxed{v = \sqrt{{v_x}^2 + {v_y}^2}}$ and arctangent $\boxed{\theta = \tan^{-1}\dfrac{v_y}{v_x}}$

The highest point in the flight: $\boxed{v_x = v_{0x}}$ and $\boxed{v_y = 0}$. If the problem ended here these conditions would apply.

Horizontal Launches

The launch angle is $\boxed{\theta = 0^o}$

$$v_{0x} = v_0 \cos\theta = v_0 \cos 0^o = v_0 \qquad \boxed{v_{0x} = v_0}$$

$$v_{0y} = v_0 \sin\theta = v_0 \sin 0^o = 0 \qquad \boxed{v_{0y} = 0}$$

The above math is not really necessary. Inspection of the Fig 3.4 shows that v_0 is directed straight down the *x*-axis with no *y*-component vector visible at all. The end of the problem is similar to the problem depicted in Fig 3.3, above.

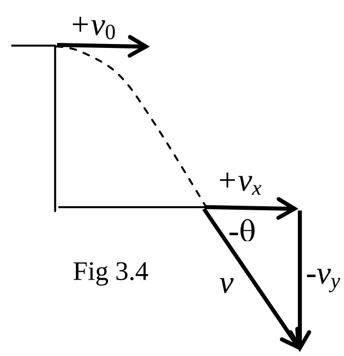
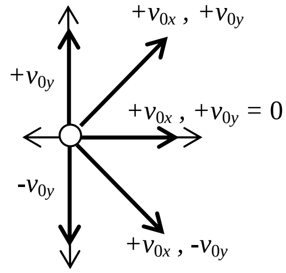
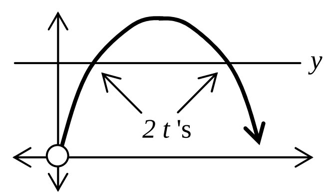
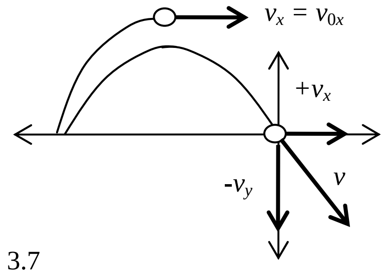


Fig 3.4

Coordinate Axis System provides the necessary orientation to handle the following variables and their appropriate signs: launch angle, initial velocities in x & y, final velocities in x & y, final landing height, and final overall velocity. Orientation matters and thus the coordinate axis becomes a powerful tool, as depicted on the next page.

Initial Launch	***During the problem***	***Final Landing***
 Fig 3.5	(At the top $v_x = v_{ox}$ and $v_y = 0$)  Fig 3.6	 Fig 3.7
Initial displacement $x_0 = 0$ $y_0 = 0$ Falling bodies: θ = ±90° $v_{0x} = v_0 \cos 90^o = 0$ $v_{0y} = v_0 \sin 90^o = v_0$ Horizontal launch: θ = 0° $v_{0x} = v_0 \cos 0^o = v_0$ $v_{0y} = v_0 \sin 0^o = 0$ 1st quadrant launch: +θ $v_{0x} = v_0 \cos\theta$ will be + $v_{0y} = v_0 \sin\theta$ will be + 4th quadrant launch –θ $v_{0x} = v_0 \cos\theta$ will be + $v_{0y} = v_0 \sin\theta$ will be –	If it lands at the same height as it started ($y = y_0$), then $t_{up} = t_{down}$. There are two t's for every y. The shorter t is for the upward trip. The longer t is for the downward trip. Solve for maximum height two ways 1. From ground up where $v_y = 0$. $v_y^2 = v_{0y}^2 + 2g\left(y - y_0\right)$ 2. Or the easy way. Start at the top and pretend it is a falling body. v_x doesn't matter since time is controlled by the y-direction. And at the top v_{oy} is zero. However, this solves for half of the total flight. $y = \frac{1}{2}gt^2$ ***<u>Must double time!</u>***	+ x Always – y Lands lower than y_0 $y = 0$ Lands same height as y_0 + y Lands higher than y_0 $a_x = 0$ No a in the x-direction $v_x = v_{0x}$ What is it doing at the end of the problem in the y-direction? $\boxed{\pm v_y}$ It is usually moving downward at the end of the problem. So v_y is ***<u>usually negative</u>*** The final $\mathbf{v}$ must be resolved. $v = \sqrt{v_x^2 + v_y^2}$ If $y = y_0$ $v_y = -v_{0y}$ & $v = v_0$

Projectile Motion Strategies

1. ***Horizontal Launch:*** Since $v_{0y} = 0$ and $v_{0x} = v_0$, then use $\boxed{y = 1/2\ gt^2}$ and $\boxed{x = v_{0x}t}$.
2. ***When time (t) or range (x) is given:*** Start with $\boxed{x = v_{0x}t}$ and then $\boxed{y = y_0 + v_{0y}t + 1/2\ gt^2}$.
3. ***No x and no t:*** Time is the key to falling body & projectile problems. Two strategies are useful when time is missing.

1st $y = y_0 + v_{0y}t + 1/2\ gt^2$	1st $\boxed{v_y^2 = v_{0y}^2 + 2g\left(y - y_0\right)}$ solve for v_y which is usually ***<u>negative</u>***.
2nd Quadratic Equation	2nd $\boxed{v_y = v_{0y} + gt}$ use -v_y from above to get t.
3rd $\boxed{x = v_{0x}t}$	3rd $\boxed{x = v_{0x}t}$ use t from above to solve for range x.

Other Projectile Motion Facts

Any ***two launch angles that add to 90°*** will arrive at the same landing site if fired on level ground. Examples: 15° and 75°, 30° and 60°, and 40° and 50°. ***Maximum range*** (maximum distance in the x-direction) is achieved by launching at 45° (45° + 45° = 90°). ***Maximum altitude*** (maximum distance in the y-direction) is achieved by firing straight up, at 90°.

Circular Motion

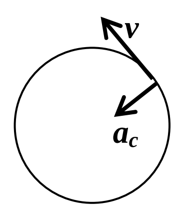


Fig 3.8

Frequency: How often a repeating event happens. Measured in revolutions per second.

Period: The time for one revolution, $\boxed{T = \frac{1}{f}}$. Time is in the numerator.

Velocity: In uniform circular motion the magnitude (speed) of the object is not changing. However, the direction is constantly changing, and this means a change in velocity (a vector composed of both magnitude and direction). In circular motion one can describe the rate of motion as either a speed or as a **tangential velocity** $\boxed{v = \frac{2\pi r}{T}}$. This velocity is an instantaneous velocity and it is directed tangent to the curve.

Centripetal Acceleration: The object is continually turning toward the center of the circle, but never gets there due to its tangential velocity. This centripetal (center seeking) change in velocity, is a centripetal acceleration $\boxed{a_c = \frac{v^2}{r}}$

1–4 Newton's Laws of Motion and Force Vectors

Force: Any push or pull.

Newton's 1st Law: Law of inertia, a restatement of Galileo's principle of inertia. Inertia is controlled by mass, the more mass the more inertia. Essentially an object at rest wants to stay at rest, while an object in motion wants to stay in motion.

Newton's 2nd Law: $\boxed{\sum F = ma}$. The Greek sigma proceeding F is the mathematical notation for taking a sum. In reality there are countless forces interacting with every object in the known universe. When an object is standing still or moving at constant velocity all of these forces are counteracting and neutralizing each other. If a single additional force is applied (***applied force***) it can upset this balance forcing the object to change its inertia and accelerate. But, there may be more than one of these applied forces acting in a physics problem. These applied vector forces need to be added before a determination of the objects acceleration can be made.

Net Force: $\boxed{\sum F = ma}$

$\sum F_x$ is $\sum F$ in the ***x***-direction on the coordinate axis.

$\sum F_y$ is $\sum F$ in the ***y***-direction on the coordinate axis.

$\sum F_{\parallel}$ is the $\sum F$ parallel to a slope.

$\sum F_{\perp}$ is the $\sum F$ perpendicular to a slope.

Newton's 3rd Law: When two entities interact there is an equal and opposite force exerted on each object. Forces come in action-reaction pairs. For every ***action force*** there is an equal & opposite ***reaction force*** (***not*** an equal and opposite reaction). The reaction that is seen also depends on the mass of the object. If a high mass object encounters a low mass object, the one with less mass appears to be effected the most (either moves radically or sustains the most damage). This is due more to its low mass, since the force on both objects is the same.

Common Forces: In addition to the forces below, $F_{\textit{any subscript that makes sense}}$, can be used.

There are many vectors in physics: displacement, velocity, acceleration, force, momentum, gravity fields, electric fields, magnetic fields, etc. The last three mentioned, gravity, electricity, and magnetism are field forces. These are normally associated with invisible force fields. When you push on a box the force is visible and obvious. Gravity pulling down on the box, toward earth, is not visible and not as obvious. These invisible forces are called field forces and motion is along the field lines. ***Often these field forces are not specifically mentioned, but they are implied by the conditions in the problem***.

$\sum F$	$\sum F = ma$	***Sum of Force*** for linear motion, not used in circular motion.
$F_1,\ F_2,\ \ldots$		***Applied Force***, some kind of push or pull.
F_g or W	$F_g = mg$	***Force of Gravity*** which is also known commonly as ***Weight***.
T		***Tension*** is the force that acts along strings, ropes, chains, etc.
N	$N = mg\cos\theta$	***Normal Force***: A contact force that exists when surfaces touch. It is always perpendicular to the surface. The angle θ refers to the angle of a sloping surface. On flat surfaces $\theta = 0^o$ and $\cos 0^o = 1$, so $N = mg$. However, this shortcut is only true as long as there are no applied forces or components of applied forces perpendicular to the surface. Additional y-forces or y-components of force can make the normal force one of the most difficult forces. See Ex. 4.4 on page 8.
f or F_{fr}	$f = \mu N$	***Force Friction***: A combination of the roughness of the surface, μ, and the amount of force pushing the surfaces against each other (the normal force), N.
F_c	$F_c = ma_c$	***Force Centripetal***: Sum of force for circular motion, not used in linear motion.
F_E	$F_E = qE$	***Force of Electricity***
F_B	$F_B = qvB$ $F_B = BI\ell$	***Force of Magnetism*** on a charge moving in a magnetic field. ***Force of Magnetism*** on a current carrying wire in a magnetic field.

From a vector stand point, it does not matter what the exact force is when solving force problems. A force is a force is a force. The variable changes, F_g, T, N, F_c, F_E, F_B, or F_1, F_2, F_3, but it is still force. And when we finish with forces and move to momentum the letter will change to p instead of F, but the vector problem solving technique remains the same.

Free Body Diagram: Imagine that you are looking down at a box from above, and that four people are pulling on ropes in the direction of the four arrows in Fig. 4.1. A diagram that just shows the object and the forces immediately acting on the object (and nothing else) is known as a free body diagram. As drawn the length of each force vector is an indication of its strength. Obviously persons 1 and 4 are stronger than 2 and 3. In free body diagrams the arrows do not have to be drawn proportionally, they just need to be pointing the correct way and be correctly labeled. The vectors are drawn coming out of the body. This allows an imaginary coordinate axis, shown in Fig. 4.1 by the dashed lines, to be superimposed through the center of the object. This is used to reference the direction of each force vector. Vector direction is simplified as + and – signs can be used to indicate direction along any axis. (In vector math the magnitudes +2 and –2 are equal. They are just opposite in direction.)

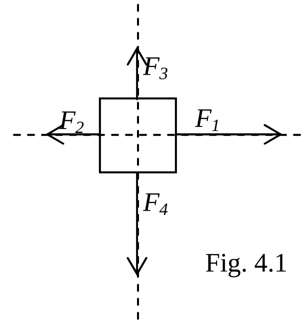


Fig. 4.1

Hints to Complete Successful Free Body Diagrams

1. Identify any invisible force fields like gravity, electricity, and magnetism. Gravity is in every problem and points down.
2. If there is a string or rope, a force of tension exists along the string or rope and it is directed away from the object.
3. If there is a surface or contact point, there is a normal force directed away from and perpendicular to the surface.
4. Applied forces are the most obvious. They are in the text of the problem and may also be in any diagrams if provided.

Finding the Force Resultant (Sum of Force) in Various Situations

Example 4-1: All Vectors Lie Along an Axis.

Look at Fig. 4.1 and suppose that F_1 = 5 N, F_2 = 2 N, F_3 = 2 N, F_4 = 5 N.

1. Draw a Free Body Diagram (FBD), as shown in Fig. 4.1.
2. Write vector sum equation in any relevant direction(s). $\sum F_x = F_1 - F_2$ $\quad \sum F_y = F_3 - F_4$
3. Substitute known equations and/or values. $\sum F_x = 5 - 2 = 3$ $\quad \sum F_y = 2 - 5 = -3$
4. Solve. $\sum F_x = 3$ $\quad \sum F_y = -3$
5. If needed: Resolve components into a resultant. $\sum F = \sqrt{F_x{}^2 + F_y{}^2} = \sqrt{(3)^2 + (-3)^2} = \boxed{4.24N}$
6. If needed: Solve for the direction. $\theta = \tan^{-1}\dfrac{y}{x} = \tan^{-1}\dfrac{(-3)}{(3)} = \boxed{-45^o}$

Example 4-2: Converting Vectors at Angles Into Components on the Coordinate Axis.

In Fig 4.2, suppose that F_1 = 5 N at 40°, F_2 = 2 N at 230°, F_3 = 2 N at 70°, F_4 = 5 N at –60°.

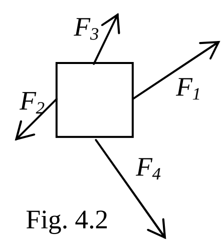


Fig. 4.2

1. Draw a FBD, as shown in Fig. 4.2.
2. State the vector sum equation in direction that matters
$$\sum F_x = F_{1x} + F_{2x} + F_{3x} + F_{4x} \qquad \sum F_y = F_{1y} + F_{2y} + F_{3y} + F_{4y}$$
3. Substitute known equations and/or values
$$\sum F_x = F_1 \cos\theta + F_2 \cos\theta + F_3 \cos\theta + F_4 \cos\theta$$
$$\sum F_y = F_1 \sin\theta + F_2 \sin\theta + F_3 \sin\theta + F_4 \sin\theta$$
$$\sum F_x = 5\cos 40^o + 2\cos 230^o + 2\cos 70^o + 5\cos -60^o \qquad \sum F_y = 5\sin 40^o + 2\sin 230^o + 2\sin 70^o + 5\sin -60^o$$
4. Solve $\sum F_x = 5.73N$ $\quad \sum F_y = -0.769N$
5. If needed: Solve for the total with Pythagorean theorem $\sum F = \sqrt{F_x{}^2 + F_y{}^2} = \sqrt{(5.73)^2 + (-0.769)^2} = \boxed{5.78N}$
6. If needed: Solve for the direction $\theta = \tan^{-1}\dfrac{y}{x} = \tan^{-1}\dfrac{(-0.769)}{(5.73)} = \boxed{-7.64^o}$

Example 4-3: Using a Flexible Coordinate Axis System.

It will be easier in more complex problems if we set the direction that an object is moving to be positive. Once this direction is declared as positive all vectors pointing that way are positive, and those in the opposite direction are negative.

x-direction:

If object is moving right then,
direction of motion is right.
Right vectors are positive.

$$\sum F_x = F_1 - F_2 - F_3$$

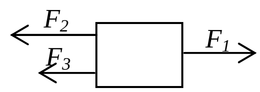


Fig 4.3a

If object is moving left, then
direction of motion is left.
Left vectors are positive.

$$\sum F_x = F_2 + F_3 - F_1$$

y-direction:

If object is moving up, then
direction of motion is up.
Up vectors are positive.

$$\sum F_x = F_1 - F_2 - F_3$$

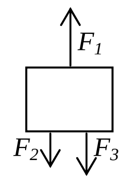


Fig 4.3b

If object is moving down, then
direction of motion is down.
Down vectors are positive.

$$\sum F_x = F_2 + F_3 - F_1$$

Note: The force vectors have already been assigned as positive and negative. So, ***plug in only positive numbers***, such as $g = +9.8$. ***Do not plug in negatives***, like, -9.8, as this will reverse the vector direction decided in the sum of force equation.

Falling Bodies and Projectile Motion Revisited: Try the above method for a falling body that is dropped at rest, or a projectile that is fired horizontally or downward. A vector is a vector, so whether it is force, velocity, or acceleration etc. use the same technique. A falling body is moving downward. So set the direction of motion as positive, thus down becomes positive. Any vector pointing down is now positive.

$y = \dfrac{1}{2}gt^2$ Both g and y point down and are positive. Everything is now positive. You are in charge of the problem. It does not matter what direction is positive. It matters that there is sign consistency. If down is defined to be positive then any vector pointing upward better have a positive. The beauty of using the direction of motion as positive and assigning plusses and minuses to the vector quantities is that you get to plug in positive values. ***(This is difficult to do in a falling body or projectile motion problem that travels in one direction(up) at the start of the problem and another direction (down) at the end of the problem. For these problems it may be easier to use the rigidly defined coordinate axis system with g as -9.8 that you have previously learned).***

Example 4-4: Deciding On The Relevant Direction.

Suppose a 10.0 kg box, in Fig. 4.4, is pulled along a flat surface by a 20.0 N force at a 30.0° angle with the horizontal. This problem is a little different than the previous ones. The x-direction and the y-direction are doing different things. The object is accelerating in the x-direction, while it is not moving in the y-direction. We can solve for the acceleration and for the normal force using the sum of forces in the relevant direction for each of these quantities.

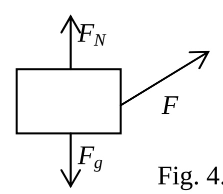


Fig. 4.4

1. Draw a FBD
2. Find the vector sum equation(s). This box will accelerate in the x-direction, so if acceleration data is requested, sum the forces in the x-direction. If the problem asks for a force in the y-direction, such as force normal, sum the forces in the y-direction.

$$\sum F_x = F_x \qquad\qquad \sum F_y = N + F_y - F_g$$

3. ***<u>Ask yourself what the object is doing in the relevant direction.</u>*** There are three choices:
   1. Standing still, $a = 0$. 2. Constant velocity, $a = 0$. 3. Accelerating, $a = a$.
   This problem is accelerating in the x-direction, while standing still in the y-direction.
4. Substitute known equations and values. *The known equations come from the table on page 6.*

$$ma_x = F\cos\theta \qquad\qquad ma_y = N + F\sin\theta - mg$$

$$10.0a_x = 20.0\cos 30^o \qquad\qquad 10.0(0) = N + 10.0\sin 30^o - (10.0)(9.8)$$

5. Solve

$$a_x = \frac{20.0\cos 30^o}{10.0} \qquad\qquad N = (10.0)(9.8) - 10.0\sin 30^o$$

$$a_x = \boxed{1.73\,m/s^2} \qquad\qquad N = \boxed{93.0N}$$

What Is The Object Doing?

It is critical to know how the object is moving.

1. Inertia: The object is following the principle of inertia. It lacks acceleration.
   a. Constant Velocity: $a = 0$, which means that $\sum F = ma = 0$.
   b. Standing Still: $a = 0$, $\sum F = ma = 0$. Standing still, $v = 0$, is just a constant velocity that happens to be zero.
2. Accelerating: $\sum F = ma$. a is solved using this equation, or by other means and then is plugged into this equation.

Example Scenario:

1. Standing still in a car: $\Delta v = 0$ $a = 0$ $\sum F = ma = 0$ Inertia.
2. Step on gas peddle: $\Delta v = +$ $a = +$ $\sum F = F_{engine} = ma = +$ Accelerates.

   Initially there is only one applied force acting in this problem, and it is the cars engine (really it is friction between tires and road). This force is in the direction of motion and is a positive force causing positive acceleration. But as soon as the car starts moving it experiences air resistance. It will continue to accelerate as long as the sum of force is positive.

   $\Delta v = +$ $a = +$ $\sum F = F_{engine} - F_{air} = ma = +$ Accelerates.
3. Reach cruising speed: $\Delta v = 0$ $a = 0$ $\sum F = F_{engine} - F_{air} = ma = 0$ Inertia.

   Why did it reach a constant velocity with the gas peddle still pressed? The car encounters air resistance, a second force. Air resistance is in the opposite direction and is negative. At first the air resistance is small, but grows until it equals the force of the engine. Adding the positive force of the engine to the now equal negative force of air resistance results in a $\sum F = ma = 0$, and inertia / constant velocity takes over. Note: equal and opposite force does not mean that the car stops. It continues doing whatever it was last doing, i.e., it moves with the final velocity from the acceleration phase.
4. Step on the brake: $\Delta v = -$ $a = -$ $\sum F = -F_{brake} - F_{air} = -ma = -$ Decelerates.

   The brakes (again, really friction) apply a force that opposes the direction of motion. Now there are two forces slowing the car. These negative forces decelerate the car.
5. Stopped: $\Delta v = 0$ $a = 0$ $\sum F = ma = 0$ Inertia.

Example 4-5: Balanced Forces

The forces parallel to the relevant direction add to a sum of force of zero.

As an example: All the forces in the Fig. 4.5 are parallel to the x-axis.

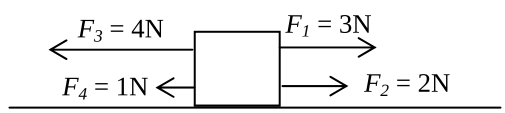


Fig. 4.5

The sum of force is: $\sum F = F_1 + F_2 - F_3 - F_4$

$\sum F = 3 + 2 - 4 - 1 = 0$

The object does not accelerate, but rather has constant velocity (which includes $v = 0$).

<u>This implies that constant velocity and standing still are the result of balanced forces, and this can be used as a shortcut.</u>

What if the problem were worded differently?

The box in Fig. 4.6 is moving at constant velocity. What is F_4 ?

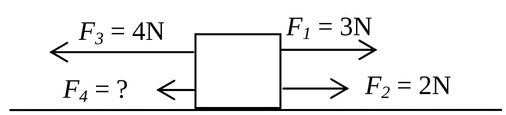


Fig. 4.6

Constant velocity implies that $\sum F = ma = 0$.

Now we know that this implies a balanced force scenario as well.

Solve using the ***sum of forces approach***.

$\sum F = F_1 + F_2 - F_3 - F_4$

$0 = F_1 + F_2 - F_3 - F_4$

$F_1 + F_2 = F_3 + F_4$

$F_4 = F_1 + F_2 - F_3$

$F_4 = 3 + 2 - 4$

$F_4 = 1$

Solving using ***balance force approach***.

All the forces in one direction (added together) must equal all the forces in the opposite direction (added together).

$F_1 + F_2 = F_3 + F_4$

$F_4 = F_1 + F_2 - F_3$

$F_4 = 3 + 2 - 4$

$F_4 = 1$

The balanced force approach is faster, as it eliminates the first two steps. ***<u>However, when an object is accelerating balanced forces do not apply, and the problem must start with a sum of forces equation.</u>***

1–5 Applications of Force

Strategy for Force Problems

1. Draw ***FBD***.
2. Decide direction of motion. Considered this the positive direction. ***If the object does not move, ask which direction it would move if it were free to do so, and set this as the positive direction.***
3. **Which direction matters, the x or the y-direction? What is it doing in the direction that matter?**
4. Construct $\sum F$ equations in the relevant direction, by looking at the ***FBD***. Any force vectors or components of force vectors pointing in the direction of motion are positive while any vectors or components opposing motion are negative.
5. Substitute known equation, $\sum F = ma$, $F_g = mg$, from the table on page 6, into the sum of force equation.
6. Plug in values and solve. All values including 9.8 are positive since the plusses and minuses have already been decided.

Example 5-1: Static Force Problems, and Force Triangles

In beginning physics many force problems contain a force triangle.

A mass is suspended by two strings from the ceiling, as shown in Fig 5.1a. Fig. 5.1b is the FBD. In Fig 5.1c the tensions are separated into x *and* y components. Sum the forces in the x and y-directions separately. The sum of force will be equal to zero.

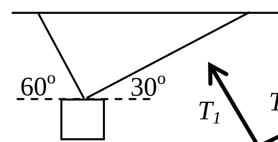


Fig 5.1a

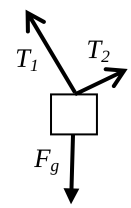


Fig 5.1b

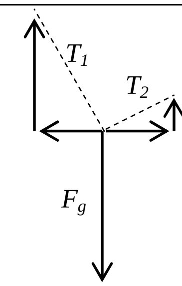


Fig 5.1c

$\sum F_x = T_{2x} - T_{1x}$ $\boxed{T_{1x} = T_{2x}}$

$\sum F_y = T_{1x} + T_{2y} - F_g$ $\boxed{T_{1x} + T_{2y} = F_g}$

Under the right circumstances the three force vectors form a right triangle when added tip to tail. Note the 30° and 60° angles above. The forces complete the 30-60-90 triangle shown in Fig 5.1d. The sum of force is zero. Tensions can be calculated using SOH CAH TOA and Pythagorean Theorem.

$\boxed{T_1 = F_g \cos 30^o}$ $\boxed{T_2 = F_g \sin 30^o}$ or $\boxed{T_1 = F_g \sin 60^o}$ $\boxed{T_2 = F_g \cos 60^o}$

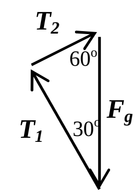


Fig 5.1d

Example 5-2: Lawn Mower

An old fashioned lawnmower is pushed with 90N at a 45° angle against a horizontal retarding force. Fig 5.2a is the FBD, while Figure 5.2b is a diagram of horizontal and vertical component vectors

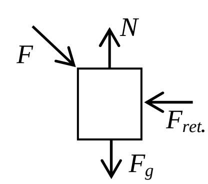


Fig 5.2a

Solve for the Retarding Force

$\sum F_x = F_x - F_{ret.}$ $0 = F_x - F_{ret.}$ $F_{ret.} = F_x = 90N \cos 45^o = \boxed{63.6N}$

Solve for the Normal Force

$\sum F_y = -F_y + N - F_g$

$N = \sum F_y + F_y + F_g$ $N = 0 + \left(90N \sin 45^o\right) + \left[\left(16kg\right)\left(9.8\,m/s^2\right)\right] = \boxed{220N}$

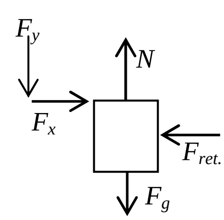


Fig 5.2b

Solve for F to accelerate from rest to 1.5 m/s in 2.5 s

$v_x = v_{x_o} + a_x t$ $a_x = \dfrac{v_x - v_{x_o}}{t} = \dfrac{1.5-0}{2.5} = 0.6\,m/s^2$ $\sum F_x = ma_x = \left(16kg\right)\left(0.6\,m/s^2\right) = \boxed{9.6N}$

You need this force to accelerate, but you still need to overcome the retarding force.

$\sum F_x = F_x - F_{ret.}$ $F_x = \sum F_x + F_{ret.} = 9.6N + 63.6N = 73.2N$

But you aren't pushing in the x-direction. You need the push at 45° to generate 73.2 N in the x-direction.

$F_x = F \cos 45^o$ $F = \dfrac{F_x}{\cos 45^o} = \dfrac{73.2N}{0.707} = \boxed{104N}$

Apparent Weight: When you ride in an elevator upward you will feel heavier when the elevator accelerates and lighter as it slows to a stop. When riding downward you will feel lighter when it accelerates and heavier when it stops. When you ride a roller coaster you experience the same sensations when moving up and down. On "Superman the Ride" you can actually feel weightless. Astronauts and pilots experience these same sensations when moving away or toward the center of the earth (or the moon). Astronauts also feel weightlessness as well. This effect is not limited to the vertical or y direction.

When you “step on it” in a car you feel yourself pressed into the seat, and when you panic stop you feel yourself thrown forward. These sensations have the same characteristics as gravity or weight. Up to now we have analyzed the motion of the car, the plane, the rocket, etc. using straight forward force and kinematics. <u>With apparent weight we are dealing with a false force that the passenger feels</u>. The effect is really created by the passenger’s inertia. When you “step on it” you don’t sink into the car seat. You actually follow inertia and stay at rest, while the car hits you from behind. The real acting forces are the opposite of what our brain thinks. ***To analyze this feeling of force for a passenger we need apparent weight***. Apparent weight is the weight that would show on a bathroom scale if you were between you and the surface causing the force. Contact with any surfaces is a normal force and bathroom scales measure normal force. So weight apparent is also F_N.

$\boxed{F_{g_{apparent}} = mg \pm ma}$ This equation adds the acceleration of a passenger’s vehicle to the real weight.

x* direction:** $F_{g_{apparent\ x}} = mg_x \pm ma_x$, but with no ***g in the ***x*** direction $\boxed{F_{g_{apparent\ x}} = \pm ma_x}$

Positive: accelerating, you feel heavier. Negative: decelerating, you feel lighter.

***y* direction:** $\boxed{F_{g_{apparent\ y}} = mg_y \pm ma_y}$, so acceleration adds / subtracts from the real / actual weight.

Positive: moving away from the center of gravity (up or away from Earth). Negative: moving toward the center of gravity.

Why do you feel weightless on “Superman the Ride”? Because, the acceleration is downward and matches gravity.

$$F_{g_{apparent\ y}} = mg_y \pm ma_y = m(9.8) - m(9.8) = 0\,m/s^2$$

g’s: The acceleration of gravity can also be expressed in g’s. $\boxed{1g = 9.8\,m/s^2}$ This is commonly used in flight terminology.

Example 5-3: Compound Bodies, One Dimension

These problems have two or more masses connected by a string or pressed against each other.

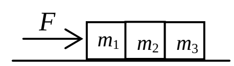


Fig 5.3a

In Fig 5.3a a force F presses against block 1 which presses with a normal force against block 2 which then presses with a normal force on block 3.

If all the blocks are the same mass what is the acceleration of the blocks? Simply treat all the masses as one larger block, and remember to use the sum of force to find acceleration.

$$\sum F_{system} = F \qquad m_{system}a = F \qquad (m_1 + m_2 + m_3)a = F \qquad \boxed{a = \frac{F}{(m_1 + m_2 + m_3)}}$$

How much force F is acting on each block? The force will distribute proportionally based on mass. If the blocks are of equal mass then the force on each of the three blocks will be one third. $\boxed{F_1 = \frac{F}{3}}$ $\boxed{F_2 = \frac{F}{3}}$ $\boxed{F_3 = \frac{F}{3}}$

If the blocks do not have the same mass then you must distribute the force using the mass ratio.

How much force is acting at the boundary between the blocks? This force is a normal force as it is created by surface contact. Block 1 requires one third of F to move. So two thirds of F remain to push blocks 3 and 4. The force at the boundary between blocks 1 and 2 is two thirds F, and this is the force needed to push the blocks behind the boundary. The last block C only requires one third of force F and thus the force at the boundary between blocks B and C is one third F. This may make more sense when examined vertically. ***If the same blocks are stacked on a table one could imagine that they are gymnasts standing on each other’s shoulders.*** How much of the total force F is felt on the bottom gymnasts feet? All of the force. How much is at the boundary? The boundary can be analyzed from either surface of contact. If we look at the bottom gymnasts shoulders, he must push up with 2/3 F to hold 2/3 m. The middle gymnasts feet must support his own weight and that of the gymnast above, so his feet must push with 2/3 F. No matter how you look at the boundary you arrive at the same answer. The boundary between the middle and top gymnast is 1/3 F, as the middle gymnast shoulder and top gymnasts feet only need to support the top gymnasts 1/3 m.

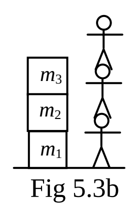


Fig 5.3b

The problem is exactly the same if the blocks are tied by strings and hanging from the ceiling. Only now we solve for tension instead of force normal, and the gymnasts are hanging from a cliff. If the top rope (top arm) in Fig 5.3c has a tension of F, what is the tension between blocks 1 and 2? 2/3 F. The top gymnast lower arm and the middle gymnasts upper arm both have to support 2/3 m. How much tension is in the rope between blocks 2 and 3? 1/3 F

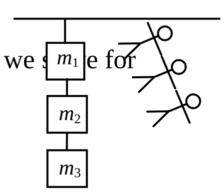
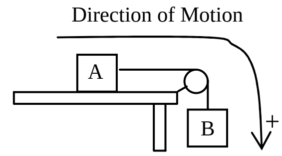
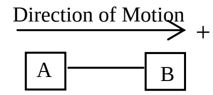


Fig 5.3c

Compound Bodies in Two Dimensions: These problems have two or more masses connected by a strings and involve pulleys. Pulleys are devices that change the direction of force. The pulleys in the following examples will be considered massless and frictionless, and as a result they do not change the magnitude of force. As two dimensions are involved simultaneously the assignment of positive and negatives on the various forces can be tricky. The easiest method to achieve sign consistency throughout a complex problem is to identify the direction the masses are moving, or the direction they are most likely to move. ***Set the direction of motion as positive***, as shown in the top figure to the right. Every vector in the direction of motion will be considered positive. Those opposed to the direction of motion are negative. If you are not sure what the direction of motion will be, take a guess. If you calculate a negative value for acceleration, you were wrong, the masses actually moved in the direction opposite your prediction. ***A very useful shortcut is to reorient the problem into one dimension.*** Pulleys are devices that change the direction of force. So pretend the pulley is not there, as shown in the figure at the bottom right.





Example 5-4: Compound Body Moving in Two Dimensions

Solve for acceleration: Fig 5.4a shows the scenario. As there are two masses there are two FBD's shown in Fig 5.4b. Fig 5.4c is an informal sketch of connected boxes. Use this sketch and the combined mass method to solve for overall acceleration. ***<u>Remember, when you use the combined method you must total all the masses for the sum of force</u>.*** F_g and F_N acting on mass A are perpendicular to motion. They cancel each other. Fig 5.4c shows that the ***<u>tension in the rope also cancels</u>***. It is the same rope so the value at both ends is the same and the direction of tension is opposite. So tension cancels in the shortcut to find acceleration.

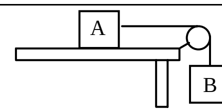


Fig 5.4a

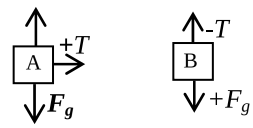


Fig 5.4b

$\sum F_{AB} = F_{gB}$ $\qquad$ $(m_A + m_B)a = m_B g$ $\qquad$ $\boxed{a = \dfrac{m_B g}{(m_A + m_B)}}$

Solve for the tension in the rope: In order to solve for tension you need a formula with tension in it. You must solve for one of the masses by itself. Solve for either body. On tests choose the easy one, this will usually be the hanging mass.

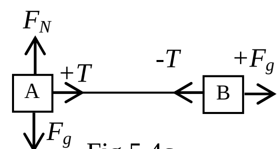


Fig 5.4c

$\sum F_A = T - F_{frA}$ $\qquad$ or $\qquad$ $\sum F_B = F_{gB} - T$

$\boxed{T = m_A a + \mu m_A g}$ $\qquad$ $\boxed{T = m_B g - m_B a}$

Plug the acceleration from part one into either equation above and you will get the same final answer.

Example 5-5: Atwood Machine:

Atwood created a device to artificially slow the acceleration of gravity. In Fig 5.4a it doesn't say which mass is greater. I picked the two masses B and C as the more massive side and used this to set the direction of motion.

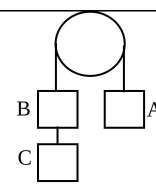


Fig 5.5a

Solve for acceleration: The FBD's for all blocks are shown in Fig 5.5b. Use the combined mass method to solve for overall acceleration. *<u>Remember, when you use the combined method you must total all the masses for the sum of force</u>*. In addition it is easier if you treat blocks B and C as though they are one larger block having a single mass. Fig 5.5c is a sketch of the masses as a linear problem, with the left masses combined.

$\sum F_{ABC} = F_{gBC} - F_{gA}$

$(m_A + m_B + m_C)a = (m_B + m_C)g - m_A g$ $\qquad$ $a = \dfrac{(m_B + m_C)g - m_A g}{(m_A + m_B + m_C)}$

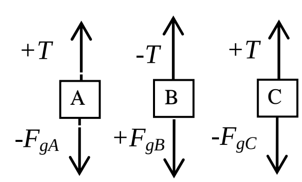


Fig 5.5b

If asked for the tension in the rope connecting mass A and B you must sum the forces for any block connected to the rope. A problem might only give information for one of the two blocks, or one of the blocks will be much simpler to solve. Learn to identify the easy block. If more than one mass is suspended by a rope, then add the masses suspended by the rope. This is the case for blocks B and C. Both of the possible solutions are detailed, one using block A on the left, and the other using blocks B and C to the right.

$\sum F_A = T - F_{gA}$ $\qquad$ or $\qquad$ $\sum F_{BC} = F_{gBC} - T$

$T = \sum F_A + F_{gA}$ $\qquad$ $T = F_{gBC} - \sum F_{BC}$

$\boxed{T = m_A a + m_A g}$ $\qquad$ $\boxed{T = (m_B + m_C)g - (m_B + m_C)a}$

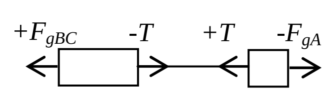
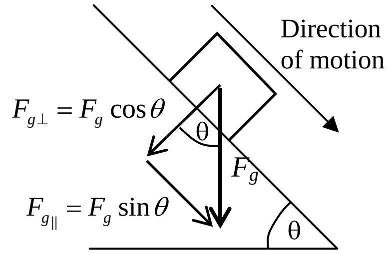


Fig 5.5c

Plug in the acceleration from above to solve for F_T. To solve for the tension in the rope between block B and C, just use the mass of block C. Refer to the right most FBD in Fig 5.5b.

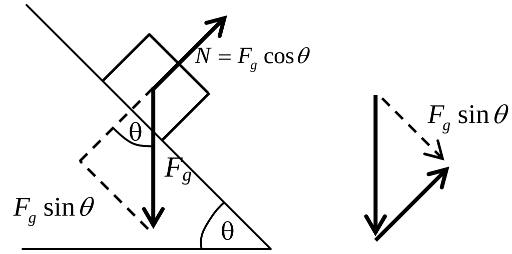
$\sum F_C = F_{gC} - T$ $\qquad$ $T = F_{gC} - \sum F_C$ $\qquad$ $\boxed{T = m_C g - m_C a}$

Force Gravity on Slopes: Motion on a slope is parallel to the slope. F_g is then at an angle to this motion. Any vector at an angle to motion should be split into components parallel and perpendicular to the chosen orientation (direction of motion). In this case F_g should split into $F_{g\,||}$ and $F_{g\perp}$ to the slope. F_g is actually pulling the block into the slope $F_{g\perp} = F_g \cos\theta$ and down the slope $F_{g_{||}} = F_g \sin\theta$.



But the block is not moving perpendicular to the slope, so the sum of forces in the perpendicular direction is zero. If gravity is pulling the block into the slope then the slope must push back with an equal and opposite force. Any force involving contact with a surface is call a **Normal Force, *N***. In slope problems $\boxed{N = F_g \cos\theta}$.

When a block moves on a flat surface the slope angle is 0°. Plug 0° into the N equation and it reduces to $\boxed{N = F_g}$. This is an abbreviation for N that works only on flat surfaces. The component of gravity parallel to the slope $\boxed{F_{g||} = F_g \sin\theta}$ is then left over. $F_{g\,||}$ is the resultant when F_g and N are added by vector addition. This is the force of gravity that can accelerate objects down slopes.



Example 5-6: Combined Bodies and Slopes

Solve for acceleration: This is just like Example 5-4, only this time block A is on a slope. Just use the same problem solving approach and recognize that gravity is different on slopes. Fig 5.7a diagrams the problem. Fig 5.7b shows the FBD's for the two masses. Make a note of the orientation of the vectors for block A. Fig 5.6c depicts the problem as a linear problem. As usual gravity is pulling block B, and as it is felt entirely in the y direction, it is acting with full strength. However, gravity is acting at an angle to the motion of block A on the slope. ***We need the component of gravity acting in the direction of motion, $F_g\sin\theta$, in order to solve the problem.***

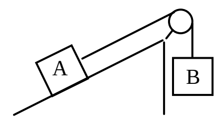


Fig 5.6a

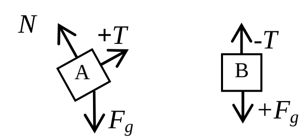


Fig 5.6b

$$\sum F_{AB} = F_{g\,B} - F_{gA}\sin\theta$$

$$(m_A + m_B)a = m_B g - m_A g\sin\theta \qquad \boxed{a = \frac{m_B g - m_A g\sin\theta}{(m_A + m_B)}}$$

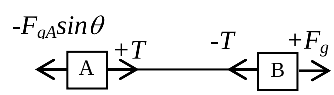


Fig 5.6c

Solve for the tension in the rope: Solve for either body. On tests choose the easy one, this will usually be the hanging mass.

$$\sum F_A = T - F_{gA}\sin\theta \qquad \text{or} \qquad \sum F_B = F_{gB} - T$$

$$\boxed{T = m_A a + m_A g\sin\theta} \qquad \boxed{T = m_B g - m_B a}$$

Plug the acceleration from part one into either equation above and you will get the same final answer.

Friction: Opposes motion and is always negative. Motion is always parallel to a surface, so friction always acts parallel.

Static Friction: Friction that will prevent an object from moving. As long as the object is standing still the force of friction must be equal to the push, pull, component of gravity or other force that attempts to move the object. (If there is no force attempting to cause motion, then there can be no friction). ***Oddly enough the maximum value for static friction is measured just as the object breaks loose and begins to move.*** *Static friction is the strongest friction since the surfaces have a stronger adherence when stationary*.

Kinetic Friction: Friction for moving objects. Once an object begins to move breaking static frictions hold, then the friction is termed kinetic. Kinetic friction is not as strong as static friction, but it still opposes motion.

Coefficient of friction: μ is a value of the adherence or strength of friction. μ_k for kinetic and μ_s for static.

$\boxed{f = \mu N}$ so $\boxed{f = \mu mg\cos\theta}$ On flat surfaces $\theta = 0^o$, $\boxed{f = \mu mg}$

Example 5-7: Two Dimension Compound Body with Friction

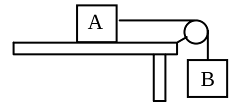


Fig 5.7a

Solve for acceleration: This is a repeat of Example 5-4, only this time friction appears in the FBD for mass A. Friction opposes motion and is therefore negative.

$$\sum F_{AB} = F_{gB} - F_{frA} \qquad (m_A + m_B)a = m_B g - \mu m_A g \qquad \boxed{a = \frac{m_B g - \mu m_A g}{(m_A + m_B)}}$$

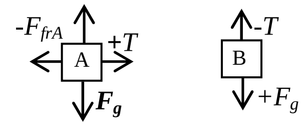


Fig 5.7b

Solve for the tension in the rope: Solve for either body. On tests choose the easy one, this will usually be the hanging mass.

$$\sum F_A = T - F_{frA} \qquad \text{or} \qquad \sum F_B = F_{gB} - T$$

$$\boxed{T = m_A a + \mu m_A g} \qquad \boxed{T = m_B g - m_B a}$$

Plug the acceleration from part one into either equation above and you will get the same final answer.

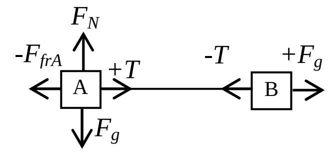


Fig 5.7c

Example 5-8: A Complex Slope Problem

Objects can move up or down a slope. $\boxed{F_g \sin\theta}$ is simply the component of gravity pulling on an object down the slope in a direction parallel to the slope. In Fig 5.8a a man is pushing mass B up a slope. In this case gravity is opposite the direction of motion, as is friction. The FBD's for both blocks are shown in Fig 5.8b, with the main difference being the presence of the true forces, F_g and F_N instead of $F_g \sin\theta$. Remember, F_g and F_N are summed to create $F_g \sin\theta$. The direction of motion (positive direction) is up the slope.

$$\sum F_A = F_{gA} - T \qquad \sum F_B = F + T - F_{gB}\sin\theta - f_B$$

$$T = F_{gA} - \sum F_A \qquad T = \sum F_B - F + F_{gB}\sin\theta + f_B$$

$$F_{gA} - \sum F_A = \sum F_B - F + F_{gB}\sin\theta + f_B$$

$$m_A g - m_A a = m_B a - F + m_B g\sin\theta + \mu m_B g\cos\theta$$

$$m_A a + m_B a = m_A g + F - m_B g\sin\theta - \mu m_B g\cos\theta$$

$$(m_A + m_B)a = m_A g + F - m_B g\sin\theta - \mu m_B g\cos\theta$$

If constant velocity $\boxed{0 = m_A g + F - m_B g\sin\theta - \mu m_B g\cos\theta}$

If accelerating $\boxed{a = \frac{m_A g + F - m_B g\sin\theta - \mu m_B g\cos\theta}{m_A + m_B}}$

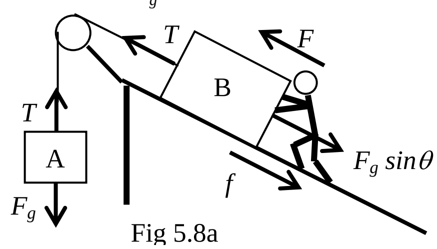


Fig 5.8a

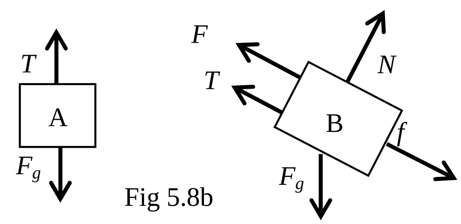
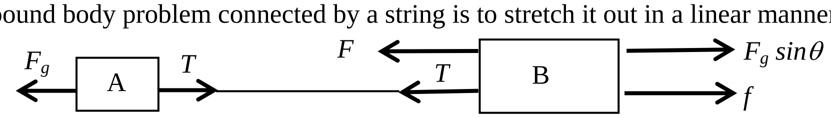


Fig 5.8b

An easier way to solve any compound body problem connected by a string is to stretch it out in a linear manner as shown in Fig 5.8c.

Fig 5.8c



The direction of motion is to the left, so all left arrows are positive, and all right arrows are negative. The string attaching the two boxes is the same and the two equal and opposite tensions cancel each other out. This allows you to solve for an overall Force Net in fewer steps. ***<u>The main difference here is that the mass in the sum of force substitution is the total mass of the whole problem</u>***.

$$\sum F = F_{gA} + F - F_{gB}\sin\theta - f_B$$

$(m_A + m_B)a = m_A g + F - m_B g\sin\theta - \mu m_B g\cos\theta$, which is identical to the sixth line in the longer version above.

Constant velocity $\boxed{0 = m_A g + F_P - m_B g\sin\theta - \mu m_B g\cos\theta}$ or accelerating $\boxed{a = \frac{m_A g + F - m_B g\sin\theta - \mu m_B g\cos\theta}{m_A + m_B}}$

However, this is not going to solve for the force of tension in the string. You must select one of the masses and work with its FBD. Block A obviously has the simpler of the FBD, so choose it.

$$\sum F_A = F_{gA} - T \qquad T = F_{gA} - \sum F_A \qquad \boxed{T = m_A g - m_A a}$$

If you are lucky the blocks are standing still or moving at constant velocity, in which case a = 0. If not use the a from above.

1–6 Work, Energy, and Power

Work: Force applied to an object that moves a distance.

$\boxed{W = \mathbf{F}\bullet\Delta\mathbf{r} = F\Delta r\cos\theta}$ θ is the angle between direction of motion and applied force.

> **Work is a Scalar (Dot) Product:** The dot product ($\mathbf{A}\cdot\mathbf{B}$) of two vectors $\mathbf{A}$ and $\mathbf{B}$ is a scalar and is equal to $AB\cos\theta$. This quantity shows how two vectors interact depending on how close to parallel the two vectors are. The magnitude of this scalar is largest when $\theta = 0^{o}$ (parallel) and when $\theta = 180^{o}$ (anti-parallel). The scalar is zero when $\theta = 90^{o}$ (perpendicular).

Work can be solved with either version of the formula. We will use both in example 6–1 below. The formula $W = \mathbf{F}\bullet\Delta\mathbf{r}$ involves vectors, which are annotated in bold print. Vectors have both magnitude and direction. The positive and negative values of force and displacement are important when using this version of the formula. The other version $W = F\Delta r\cos\theta$ involves italicized print indicating that only the magnitude of each vector is needed. Only positive numbers are used. The angle in the formula is the angle measured between the two vectors. In this version of the formula $\cos\theta$ solves the directional aspect. Compare both methods in the example below.

Example 6-1: Various Orientations of Force and Displacement.

In the first three scenarios a force $F = 5$ N acts on a mass which is displaced $r = 2$ m.

1. Force vector is parallel to the displacement vector and points in the same direction:

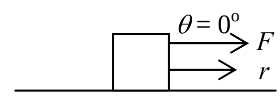


Fig 6.1a

$W = \mathbf{F}\bullet\Delta\mathbf{r}$	$W = F\Delta r\cos\theta$
$W = (+5)(2)$	$W = (5)(2)\cos 0^{o}$
We decided F was positive	Here the angle solves for the positive
$W = (+5)(2) = \boxed{10\,\text{J}}$	$W = (5)(2)(+1) = \boxed{10\,\text{J}}$

2. Force vector is parallel to the displacement vector and points in the opposite direction:

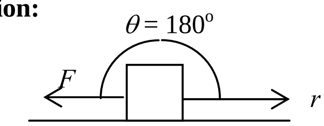


Fig 6.1b

$W = \mathbf{F}\bullet\Delta\mathbf{r}$	$W = F\Delta r\cos\theta$
$W = (-5)(2)$	$W = (5)(2)\cos 180^{o}$
We decided F is negative	Here the angle solves for the negative
$W = (-5)(2) = \boxed{-10\,\text{J}}$	$W = (5)(2)(-1) = \boxed{-10\,\text{J}}$

Forces opposing motion are negative, and are associated with negative acceleration and negative work.

3. Force and displacement vectors are perpendicular:

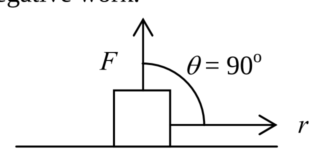


Fig 6.1c

$W = \mathbf{F}\bullet\Delta\mathbf{r}$	$W = F\Delta r\cos\theta$
$W = (0)(2)$	$W = (5)(2)\cos 90^{o}$
We decided F has no affect on motion	Here the angle solves for the zero
$W = (0)(2) = \boxed{0\,\text{J}}$	$W = (5)(2)(0) = \boxed{0\,\text{J}}$

Forces acting perpendicular to motion have no affect in the direction of the original motion. They do not speed up or slow the object in the direction being investigated. The object experiences inertia (stays at rest or continues at constant velocity) in the direction it was originally moving. No work is done in the original direction of motion. The force may accelerate the object in the perpendicular direction. However, this will no affect the direction of motion.

4. Force and displacement vectors are at angles other than parallel or perpendicular:

In this scenario $F = 5$ N at 37^{o} and $r = 2$ m

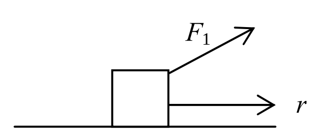


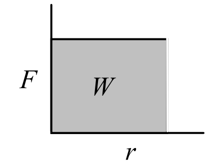
Fig 6.1d

$W = \mathbf{F}\bullet\Delta\mathbf{r}$	$W = F\Delta r\cos\theta$
To use this formula you need a component of F parallel to r	
$W = F_x r$	$W = (5)(2)\cos 37^{o}$
$W = (5\cos 37^{o})(2) = \boxed{8\,\text{J}}$	$W = (5)(2)(0.8) = \boxed{8\,\text{J}}$

Both methods result in the same calculations. They are just derived with slight differences in problem solving logic. I prefer using the formula on the left. I am in the habit of searching out my own components anyway. I know that any components perpendicular do not matter, since W = 0 in these cases. So I look for components of force that match the displacement. You can also look for components of displacement that match force. I prefer finding the correct component, since the formula on the right requires a specific angle that is not always the given angle in the problem. By solving for components I take control of the problem and avoid plugging in wrong angles.

Work is the Area Under the Force Displacement Curve

This is the integral of the force distance function in the calculus based course. In the non-calculus course these areas will be simple enough (squares, rectangles, and triangles) to allow us to use geometry to find the area. AP Physics C students need to be ready to deal with complex curves using the calculus expressions below.



Calculus: $\boxed{W = \int F \; dr}$ If the integral of force over an interval of displacement is work, then it should follow that the derivative of work with respect to the same interval of displacement is force. $F = \frac{dW}{dr}$

Energy: A capacity an object has to do work. There are many forms of energy including, mechanical, chemical, electrical, thermal, nuclear, etc. An object may have many types and amounts of energy at the start of a problem. Calculating the total energy an object has is impossible, and it is also unnecessary. During the course of a problem the type and amount of energy, or both, may change, and usually this change is restricted to only a few forms of energy. Energy can also be passed from object to object. Instead of trying to find the total energy we will focus on the forms of energy that change or that transfer from object to object. Tracking the change or movement of energy is very manageable, flexible, and extremely useful.

Mechanical Energy: The sum of Kinetic and Potential (gravitational) Energies

Kinetic Energy: Energy of motion. Depends on mass (inertia of the object) and velocity. Velocity has a greater effect on kinetic energy as it is squared in the formula $\boxed{K = \frac{1}{2}mv^2}$. Double a cars mass and you double kinetic energy. Double a cars velocity and you quadruple kinetic energy.

Potential Energy (gravitational): Energy of position. Depends on an objects mass, the gravity pulling the object, and on the height that the object is located at $\boxed{U_g = mgh}$. For mathematical simplicity and convenience the lowest possible point that the object can reach is designated to have zero height, and h is measured from this point. This is arbitrary and any point can be chosen, but choosing the lowest point as zero avoids dealing with negative heights.

Work Revisited: Think of work as the energy that is added (+***W***) to the system or subtracted (-***W***) from the system.

System: The object that the problem is focusing on.

Environment: The surrounding. The entire universe, except the object in the problem.

Work Energy Theorem: $\boxed{W = \Delta Energy}$ Work put into a system equals the change in energy of the system.

Example 6-2: Work and Potential Energy

Lifting a Mass: In order to lift a mass at constant velocity a force must be directed upward and be equal to the force of gravity. Use $W = F\Delta r \cos\theta$. Substitute F_g for F and height h for r. $W = F_g h \cos\theta$. The force and displacement are in the same direction, so $\theta = 0^\circ$ and thus $W = F_g h$. Substitution leads to $W = mgh$. Strangely this is the formula for gravitational potential energy? $U_g = mgh$. So does $W = U_g$?

Not really. Work-Energy Theorem states that work is a change in energy.

$$W = \Delta Energy \qquad W = \Delta\left(mgh\right) \qquad W = mgh_f - mgh_i$$

But if we set the lowest height in the problem (we lifted the mass so h_i is the lowest point) to be zero, then $W = mgh_f$.

So, if we arbitrarily identify the lowest height as zero, then the work done to raise an object does equal the energy it has at its new position, relative to its starting point. We are doing positive work, and this adds to the energy of the object.

If we lift an object by doing 20 J of work, then the object will have 20 J of additional energy. ***If we pretend that the object had 0 J at the start of the problem***, then it now has 20 J at the end. Caution: No object ever has zero energy. This is just a mathematical trick or simplification. We are not concerned with the amount of total energy the object has. Instead we are focusing on the change in energy. Many forms of energy are present (chemical, electrical, nuclear, etc.), but they did not change in this problem. The only energy that changed was the energy of height or position, known as potential energy. For convenience we set this energy to zero at the start of the problem (we simply moved the number line). Besides avoiding negative heights it allows easier analysis of useable energy and energy flow. The work done to lift the mass is added to the mass and becomes the final energy of the mass. This is really just the added energy, and since the object cannot fall any further than the lowest height (zero potential energy), it can only loose this much potential energy. ***We are really just tracking changes in usable energy.*** This is why Work and Work-Energy Theorem is such a useful entity.

20 J of work were added to the system, doing positive work and raising the systems energy. Where did it come from? It came from the environment. We will soon learn that energy is conserved and just either moves (transfers) between locations (system and environment) or it changes form (potential to kinetic, etc.). If you drop the object it will loose 20 J of potential energy. Since it is loosing energy, 20 J of negative work is done by gravity. Where does this energy go? It turns into 20 J of kinetic energy. Then the object hits the table and stops. Now it has lost all of its height (potential energy) and all of its velocity (kinetic energy). Where did the 20 J go? There was a sound, so molecules of air were pushed aside. The kinetic energy of the molecules increased. The table vibrated. The kinetic energy of the molecules in the table increased. This vibration is felt as heat. Where are the air molecules and table molecules? They are in the environment. So the 20 J returned to the environment. What do you mean by returned to the environment? Well, when I picked up the mass in the first place I was part of the environment, so the original 20 J used to lift the mass came from the environment in the very beginning.

Conservation of Energy: Energy cannot be created or destroyed, but it can change forms.

There are many forms of energy. Those that will be used in this course are shown in the energy wheel below (Note: AP Physics C students will not cover Thermal or Modern). All the forms of energy are included here as this serves as a review of the entire year. If any energy is unfamiliar to you at this point in the year don't worry, we will cover them soon enough.

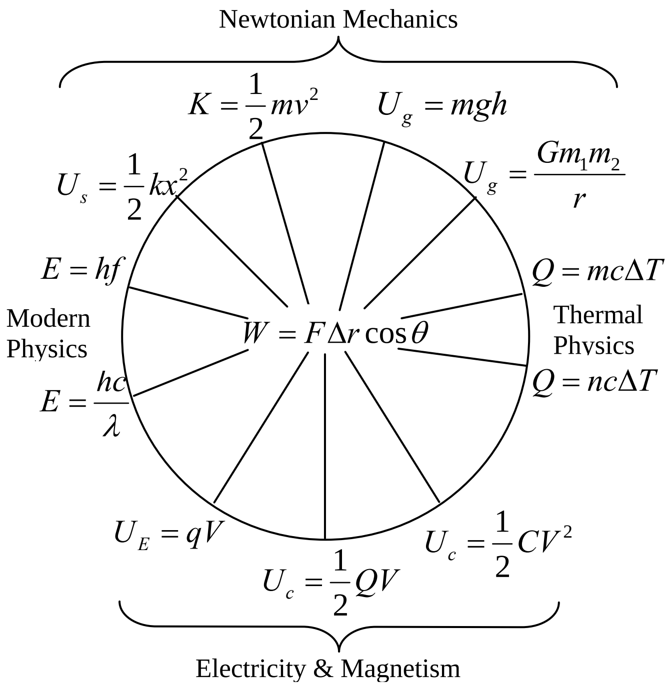
If energy can change form, then any energy can be equal to any other energy. If you drop an object the object looses height, but gains velocity. In this case potential energy is turning into kinetic energy. If the object looses all of its height, then there is a 100% transfer of energy to kinetic energy. $\boxed{mgh_i = \frac{1}{2}mv_f^{\,2}}$. But, what if the energy is an incomplete transfer. What if the problem ends before reaching zero height (zero potential energy). The object will then have two energies at the end. $\boxed{mgh_i = mgh_f + \frac{1}{2}mv_f^{\,2}}$. Energy is a scalar. It is directionless and simply adds without worrying about direction.

Basically, conservation of energy means that the total energy at the beginning of the problem must equal the total energy at the end of the problem. What if the object has height and is moving at the beginning of the problem, and still has height and is moving at the end of the problem? $\boxed{mgh_i + \frac{1}{2}mv_i^{\,2} = mgh_f + \frac{1}{2}mv_f^{\,2}}$. The total energy in the problem is either $\boxed{E_{total} = mgh_i + \frac{1}{2}mv_i^{\,2}}$ or $\boxed{E_{total} = mgh_f + \frac{1}{2}mv_f^{\,2}}$. The total energy is conserved, so the total energy is present at the beginning, and it is still present at the end. What happens if the object is thrown straight upward, from the lowest point, and then reaches the highest point of flight where the velocity is zero? $\boxed{\frac{1}{2}mv_i^{\,2} = mgh_f}$. What if it is thrown upward at an angle so that when it reaches its highest point it still has some velocity in the x- direction? $\boxed{mgh_i = mgh_f + \frac{1}{2}mv_f^{\,2}}$. We can see that conservation of energy is a very flexible and powerful tool.

Remember: Energy is directionless. Simply ask yourself:

1. What energy/energies are present initially and add them up on the left.
2. What energy/energies are present finally and add them up on the right.

Can energy be lost? No! Lost energy goes to the environment. A car (system) looses energy due to air resistance, so air molecules (environment) gain energy and move faster. Energy is conserved in the universe.



The wheel to the left shows all the energies learned in this class around the outside. They are grouped into the five major strands of physics. In the center is work.

Conservation of Energy: Energy cannot be created or destroyed it can change forms.

Changing forms means that any energy/energies initial around the outside of the circle can change into any other energy/energies final. $\boxed{\sum Energy_i = \sum Energy_f}$

In this class we often assume 100% energy transfer. One energy may be present initially, which disappears, and another energy is present at the end of the problem. In these 100% energy transfer problems any energy around the wheel can equal any other energy. If more than two energies are mentioned then you simply add up all energies present initially and set them equal to the total of all the final energies.

Work Energy Theorem: Work is a change in any energy. $\boxed{W = Any\ Energy_f - Any\ Energy_i}$

Most of the time one of the energies, either final or initial is equal to zero. So then Work can equal any energy around the outside. And since $W = Fr$, then Fr can equal any energy around the outside.

When do we use Conservation of Energy as Opposed to Work Energy Theorem? Conservation of energy is used when energy stays in the system and simply changes form. Work Energy Theorem is used when energy moves from the environment to the system and vice versa. You will see me use a hybrid of the two, in the examples below, when energy is lost to the environment due to friction.

The following is an incomplete list of examples. Many of these are common problems, and have been represented in AP Exams. In addition many are based on 100% efficiency of energy transfer. This is not really the case (see thermodynamics, soon), but it makes the problems easier in the same way that an airless and frictionless world helped us to start kinematics.

Example 6-3: Height Turning Into Velocity

The path does not matter, only the change in height.

For a 100% transfer: $\boxed{mgh_i = \frac{1}{2}m{v_f}^2}$

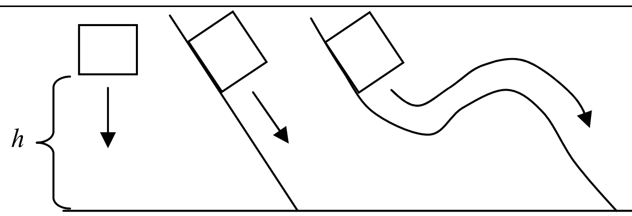


Fig 6.3

Example 6-4: Roller Coaster

A roller coaster is a good example of incomplete transfer of energy. At different points along the track there are various amounts of both kinetic and potential energy. One key to all these height problems, measured from a planetary surface, is to declare the lowest point in a problem to be $h = 0$. This gives a reference that is easy to add or subtract from. Then if you are given the height at any point on the track you can find the carts velocity:

$$\boxed{mgh_i + \frac{1}{2}m{v_i}^2 = mgh_f + \frac{1}{2}m{v_f}^2}$$

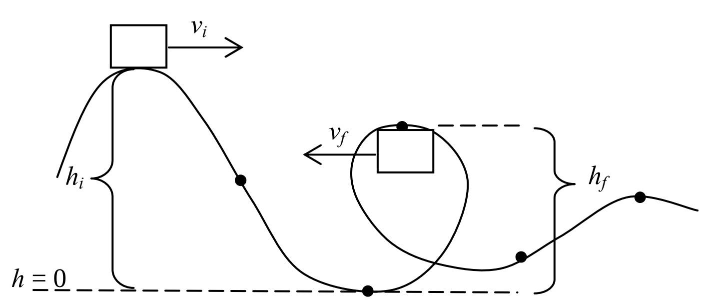


Fig 6.4

Example 6-5: Friction Stops an Object

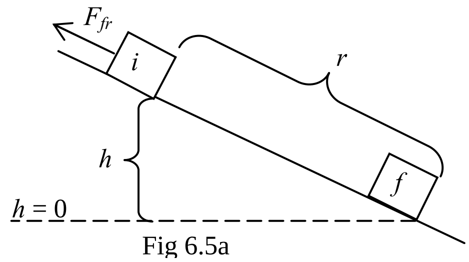


Fig 6.5a

Fig 6.3a shows an object that had height and stopped due to friction. Friction is a force and the only way to use a force in an energy equation is to multiply the force by the displacement, in other words use work. In this case it is the work of friction.

$$mgh_i = W_{fr} \qquad \boxed{mgh_i = F_{fr}r}$$

If the object in Fig 6.3a did not stop, but was just slowed by friction then it would still have some kinetic energy.

$$mgh_i = W_{fr} + \frac{1}{2}mv_f^{\ 2} \qquad \boxed{mgh_i = F_{fr}r + \frac{1}{2}mv_f^{\ 2}}$$

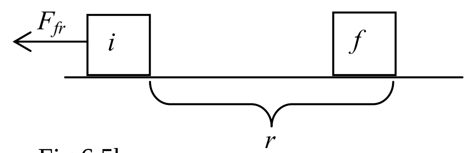


Fig 6.5b

In figure 6.3b the object is sliding along a horizontal surface. It initially has kinetic energy and is stopped by friction.

$$\frac{1}{2}mv_i^{\ 2} = W_{fr} \qquad \boxed{\frac{1}{2}mv_i^{\ 2} = F_{fr}r}$$

If the object in Fig 6.3b did not stop, but was just slowed by friction then it would still have some kinetic energy.

$$\frac{1}{2}mv_i^{\ 2} = W_{fr} + \frac{1}{2}mv_f^{\ 2} \qquad \boxed{\frac{1}{2}mv_i^{\ 2} = F_{fr}r + \frac{1}{2}mv_f^{\ 2}}$$

Power: Power is the rate of work, or rate of energy change. In other words it is the rate that energy is used, transferred, or generated during a one second interval. Since it involves energy, power is by extension as important.

$\boxed{P = \dfrac{W}{t}}$ you can substitute for work $P = \dfrac{F\Delta r}{t}$, and if you note that displacement over time is velocity then,

$\boxed{P = Fv}$. The first boxed equation is useful when you have work or energy, the second is useful when you have force. Even though the first equation contains the expression for work, you must be flexible. You must realize that work is a change in any energy $\boxed{P = \dfrac{\Delta Any\ \ Energy}{t}}$. You can plug in any energy from the preceding page.

$\boxed{P = \dfrac{mgh}{t}}$, $\boxed{P = \dfrac{1/2mv^2}{t}}$, etc. ***<u>If a problem contains any form of energy or work or the units of joules, and any quantity of time or any units of time, then it will involve power.</u>***

If energy is flexible then so is power. Occasionally a question gives power as a variable, but you need energy to solve the problem. Simply set the time equal to one second, then $P = \dfrac{W}{1s}$ and work/energy will have the same numerical value (but different units) as power for that one second. Use this value for Work/Energy to solve the problem. Just remember that all answers obtained in the problem are based on one second. If time is given later on, just multiple the energy of "one second" by the number of seconds and you've got your answer.

Powerful machines do more work in the same time, or the same work in less time.

Calculus: $\boxed{P = \dfrac{dW}{dt}}$ Power is another rate (function of time) and is therefore a derivative expression. Integrating power during a time interval will return work or energy values. $\boxed{W = \int P\ dt}$

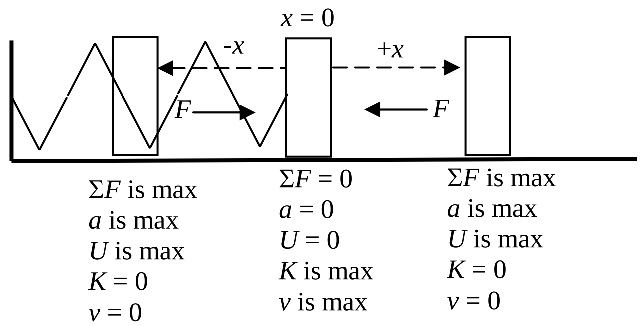
<u>Solve problems by looking for energy, work, and power first, then force, last of all kinematics.</u>

1–7 Oscillations

Period: $\boxed{T = \frac{1}{f}}$ Time for one revolution.

Frequency: The number of revolution, vibrations, oscillations, or rotations per second.

Springs: A Simple Harmonic Oscillator (SHO)

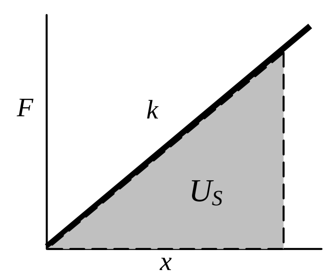


Restoring force: A spring attached to a block is shown at the right. Equilibrium ($x = 0$) is the rest position when no external forces are applied. If the spring is compressed so the block moves to $-x$ or the spring is stretched so the block moves to $+x$ the spring will have a force directed toward the equilibrium position. This is the restoring force $\boxed{F = -kx}$, and is known as Hooke's Law. ***k*** is the spring constant reflecting the quality or strength of the spring. The minus sign shows that the restoring force is opposite the displacement. Move a spring right and it wants to restore to the left. Restoring force is highest at maximum displacement (amplitude), located at $+x$ and $-x$. At these positions the spring has the highest force and acceleration, but has an instantaneous velocity of zero. It also has the highest amount of spring potential energy.

Potential Energy: The energy of position. This time we are working with a spring's position, $\boxed{U_s = \frac{1}{2}kx^2}$. When a spring is at equilibrium it is at rest and has zero displacement. This position is thus has zero potential energy. This is just like gravitational potential energy. When an object is a rest on the ground it is said to have zero potential energy. When it is displaced against the force of gravity it gains potential energy. It wants to return to the ground. The mass is displaced against the force of the spring, which wants to return it to equilibrium. So a displaced spring has stored energy, also known as potential energy. When the spring is fully displaced at either $-x$ or $+x$ it comes to rest.

Amplitude: The distance from equilibrium to the point of maximum displacement. This is the distance form $x = 0$ to $-x$, or from $x = 0$ to $+x$. It is not the distance from $-x$ to $+x$. That distance is two amplitudes. ***How many amplitudes does a spring go through in one complete cycle, from -x to x = 0 to +x, and then back to x = 0 and finally back to -x? Four.***

When the spring reaches $-x$ or $+x$ there it comes to an instantaneous stop, so it has no kinetic energy, but it has potential energy and a very large restoring force. When it moves through the equilibrium position the force on the spring and acceleration are both zero, but the velocity and kinetic energy is now at a maximum. Here any potential energy stored by displacing the spring from rest is turned into kinetic energy. Conservation of energy dictates that if one form of energy disappears and another appears, then the two energies must be equal. So $U_S = K$, or $\boxed{\frac{1}{2}kx^2{}_{\text{at either end}} = \frac{1}{2}mv^2{}_{\text{at the middle}}}$

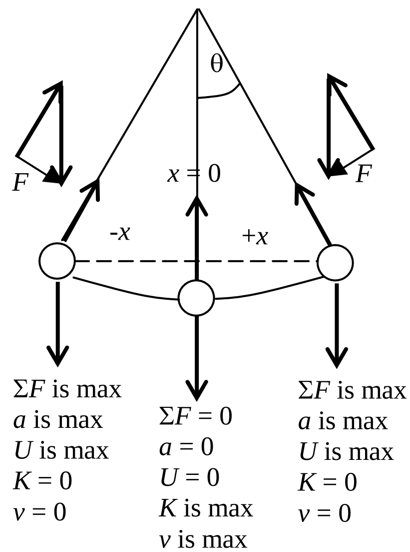


Springs are a variable force. $F_s = -kx$ The more a spring is displaced, x, the larger the force, F, that is needed to displace it. At first no force is needed to move the spring. At the end of the problem a force F_S is needed. So the average force needed during the entire displacement is $\bar{F}_s = \frac{F_s - 0}{2} = \frac{1}{2}F_s = \frac{1}{2}kx$. Work is $W = F \cdot d = \frac{1}{2}kx \cdot x = \boxed{\frac{1}{2}kx^2}$. Force and displacement can be graphed as shown at the right. The equation $F_s = kx$ is the equation for a line $y = mx + b$, where k is the slope.

U_S is the area under the curve, $\boxed{\frac{1}{2}bh} = \frac{1}{2}(x)(F_s) = \frac{1}{2}(x)(kx) = \boxed{\frac{1}{2}kx^2 = U_s}$

Period of a spring: Time required for one oscillation, $\boxed{T_s = 2\pi\sqrt{\frac{m}{k}}}$. Depends on mass of object attached to spring and k.

Pendulums: Approximates a Simple Harmonic Oscillator (SHO)



Restoring force: The restoring force is the sum of the tension and gravity vectors diagramed to the right. When the object is at its extreme displacement ($+x$ or $-x$) the restoring force is greatest. When the object returns to the equilibrium position ($x = 0$) the sum of the vectors is zero, and there is no restoring force. This is just like the spring above. There is never any net force at the equilibrium position ($x = 0$), so there is no acceleration at this point. Both the pendulum and the spring move through the equilibrium by inertia. It is interesting to note that when the restoring force is highest (at maximum displacement, either $+x$ or $-x$), the velocity is zero. When the net force is zero (equilibrium position, $x = 0$), the velocity is at its highest. All of the proceeding is true for any harmonic oscillators. Note: the dashed displacement line is not the same length as the arc that a pendulum actually moves through. The difference is very small at angles less than or equal to 15^o. Beyond this angle pendulums are not good harmonic oscillators. So at angles under 15^o pendulums behave almost like a true SHO.

Energy: The pendulum has potential energy at $+x$ and $-x$. It has kinetic energy at $x = 0$. The potential energy at one either end equals the kinetic energy in the middle. Potential energy in a pendulum has to do with height, not the spring constant $\boxed{mgh_{\text{at either end}} = \frac{1}{2}mv^2{}_{\text{at the middle}}}$

Period: Time required for one oscillation, $\boxed{T_p = 2\pi\sqrt{\frac{\ell}{g}}}$. Depends on length of the pendulum and $\boldsymbol{g}$.

1–8 Linear Momentum and Collisions

Momentum: $\boxed{\mathbf{p} = m\mathbf{v}}$ Quantity of motion (inertia in motion). Measure of how difficult it is to stop an object.

Impulse: $\boxed{\mathbf{J} = \mathbf{F}\Delta t = \Delta \mathbf{p}}$ Trade off between time taken to stop and force needed to stop. Velocity, acceleration, and momentum were understood early on in the developing days of physics. These concepts were used by Newton to establish his 2nd Law: $\mathbf{F}\Delta t = \Delta \mathbf{p}$ $\quad \mathbf{F}\Delta t = m\Delta \mathbf{v}$ $\quad \mathbf{F} = m\dfrac{\Delta \mathbf{v}}{\Delta t}$ $\quad \mathbf{F} = m\mathbf{a}$

Calculus: Force is the derivative of momentum $\boxed{\mathbf{F} = \dfrac{d\mathbf{p}}{dt}}$. Analogous to acceleration being the derivative of velocity $\boxed{\mathbf{a} = \dfrac{d\mathbf{v}}{dt}}$. If you divide the force and momentum vectors by the scalar mass you get the acceleration and velocity vectors.

Physics is full of meaningful patterns. Impulse is the integral of force during a time interval $\boxed{\mathbf{J} = \int \mathbf{F}\ dt = \Delta \mathbf{p}}$. This means that if you take the derivative of impulse with respect to time you will calculate force $\boxed{\mathbf{F} = \dfrac{d\mathbf{J}}{dt}}$.

Collisions and Conservation of Momentum: Momentum in any situation must ***always*** be conserved. When two objects, each having momentum, collide the total momentum during the problem remains the same. If mass 1 and mass 2 are moving then they both a momentum. Their momentums add together to calculate a total momentum $\boxed{p_{total} = m_1 v_{1i} + m_2 v_{2i}}$. If they collide they might bounce off each other with different velocities than before, but the total momentum must remain the same $\boxed{p_{total} = m_1 v_{1f} + m_2 v_{2f}}$. If the initial momentums and final momentums are both equal to the same total momentum then the sum of the initial momentums must equal the sum of the final momentums $\boxed{m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}}$. If there are more than two masses involved, just keep adding them to both sides.

Momentum is a Vector: Unlike energy (a scalar) which simply adds, momentum is a vector. Vectors have direction, and this means that the vector directions (like force vectors) must be accounted for in the math. You must decide on a positive direction in a conservation of momentum problem. Once decided any mass traveling in that direction has a positive momentum. Any mass traveling opposite the chosen direction has a negative momentum. Any mass traveling at an angle to the chosen direction must be split into components, with the x and y directions analyzed separately. Use the same strategies that were learned in forces.

Elastic Collision: Collisions in which kinetic energy is conserved. This can only happen when two objects do not touch each other. One example that may make sense and will be used latter in electricity is the collision between two protons. Protons have positive charges and in electricity like charges repel. If two protons approach each other head on the repulsion for each other will slow them to a stop before they touch one another. Then the repulsion will repel them away from each other. In effect they bounce off each other without touching

Inelastic Collisions: Collisions in which kinetic energy is lost. Since energy is never really lost, it must go somewhere. Lost energy is just energy that was lost by the system to the environment or it is energy that changed into a form that is not very recognizable. When masses collide and touch each other the masses vibrate. This vibration is heat. In collisions where objects touch each other some of the original kinetic energy is lost as heat.

Perfectly Inelastic Collision: The objects collide and stick together (one mass, one velocity) $\boxed{m_1 v_{1i} + m_2 v_{2i} = v_f\left(m_1 + m_2\right)}$.

Explosion: The opposite of a perfectly inelastic collision. A single object fractures and sends fragments in many directions. If we look at the simpliest case where it fractures into two bodies moving in opposite directions then $v_i\left(m_1 + m_2\right) = m_1 v_{1f} + m_2 v_{2f}$. Usually the original object is considered stationary in beginners examples, but it does not have to be $0 = m_1 v_{1f} + m_2 v_{2f}$. So this means that $\boxed{m_1 v_{1f} = -m_2 v_{2f}}$. The negative sign implies that one object must move in the opposite direction of the other.

Example 8-1: Conservation of Linear Momentum

Elastic Collision: Mass 1, m_1 = 2.00 kg, is moving at 4.00 m/s to the right. Mass 2, m_2 = 2.00 kg, is stationary and is hit by mass 1. After the collision mass 2 moves to the right at 4.00 m/s. The collision is diagrammed before and after in Fig 8.1a. What is the velocity of mass 1 after the collision?

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$(2)(4)+(2)(0)=(2)v_{1f}+(2)(4) \qquad \boxed{v_{1f}=0 m/s}$$

All the momentum from mass 1 was transferred to mass 2.

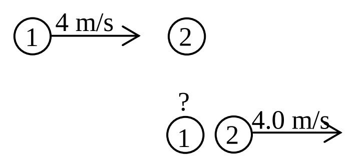


Fig 8.1a

Inelastic Collision: Mass 1, m_1 = 4.00 kg, is moving at 4.00 m/s to the right. Mass 2, m_2 = 2.00 kg, is stationary and is hit by mass 1. After the collision mass 2 moves to the right at 5.30 m/s. The collision is diagrammed before and after in Fig 8.1b. What is the velocity of mass 1 after the collision?

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$(4)(4.00)+(2)(0)=(4)v_{1f}+(1)(5.30) \qquad \boxed{v_{1f}=1.35 m/s}$$

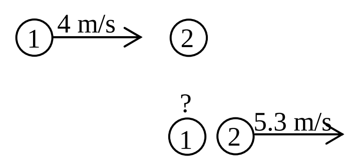


Fig 8.1b

Perfectly Inelastic Collision: Mass 1, m_1 = 4.00 kg, is moving at 4.00 m/s to the right. Mass 2, m_2 = 2.00 kg, is stationary and is hit by mass 1. After the collision the mass stick together. The collision is diagrammed before and after in Fig 8.1c. What is the velocity of the combined mass after the collision?

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$$

$$(4)(4.00)+(2)(0)=(4+2)v_f \qquad \boxed{v_{1f}=2.67 m/s}$$

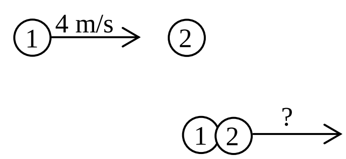


Fig 8.1c

Explosion: A large mass fractures into two smaller masses, m_1 = 4.00 kg and m_2 = 2.00 kg. Mass 2 moves to the right at 2.0 m/s. How fast is mass 1 going after the explosion?

$$(m_1 + m_2) v_i = m_1 v_{1f} + m_2 v_{2f}$$

$$(4+2)(0)=(4)v_{1f}+(2)(2) \qquad \boxed{v_{1f}=-1 m/s}$$

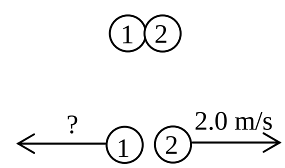


Fig 8.1d

<u>The minus sign in the last example means that the mass is going to the left. Remember to watch the minus signs. Harder problems will have masses moving in different directions. Missing the sign convention will destroy the problem.</u>

Collisions and Energy: **Total energy is always conserved**, $E_{1i} + E_{2i} = E_{1f} + E_{2f}$.

However, kinetic energy is only one form of energy. If we look only at kinetic energy it sometime seems to disappear.

Kinetic energy is conserved only in <u>elastic collisions</u>. $\qquad K_{1i} + K_{2i} = K_{1f} + K_{2f}$

Kinetic energy is lost (dissipated) in <u>inelastic collisions</u>. $\qquad (K_{1i} + K_{2i}) - K_{lost} = (K_{1f} + K_{2f})$

But where does the lost kinetic energy go? When two or more bodies collide the molecules that they are composed of are set into faster vibration. The speed of molecules is directly proportional to temperature, and temperature is one component of ***<u>internal energy</u>***. So while kinetic energy is lost, total energy is not. The kinetic energy lost is turning into another form of energy, which is not one of the ***<u>mechanical energies</u>*** (kinetic and potential). It is now in the form of vibrating atoms and molecules: ***<u>internal energy</u>*** (a thermal energy).

Example 8-2: Two dimensional Collision

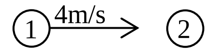


Fig 8.2a

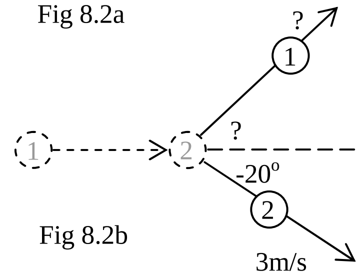


Fig 8.2b

Mass 1, m_1 = 2kg, is moving at 4m/s to the right. Mass 2, m_2 = 1kg, is stationary and is hit by mass 1 just a little off center. This causes mass 2 to move a 3m/s at an angle of 20° below the x-axis. The collision is diagrammed in Fig 8.2a (initial conditions) and in Fig. 8.2b (final conditions). What is the speed and direction of mass 1 after the collision?

The total momentum before the collision must equal the total momentum after. But after the collision the masses are moving at angles to the initial motion of m_1. We have to work in components. The total momentum in the x-direction must be conserved and the total momentum in the y-direction must be conserved.

***x*-direction**: Two bodies before and two after.

$$m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx}$$

$$(2)(4) + (1)(0) = (2) v_{1fx} + (1)\left(3\cos\left(-20^o\right)\right) \qquad v_{1fx} = 2.59 m/s$$

***y*-direction**: This dimension is an explosion. Initially there is no motion in the y-direction at all. Then there are two bodies moving in opposite directions.

$$(m_1 + m_2) v_{iy} = m_1 v_{1fy} + m_2 v_{2fy}$$

$$0 = (2) v_{1fx} + (1)\left(3\sin\left(-20^o\right)\right) \qquad v_{1fy} = 0.513 m/s$$

When you have two component vectors Pythagorean Theorem them back together and use arctangent to find the angle.

$$v_f = \sqrt{v_{1fx}^{\ 2} + v_{1fy}^{\ 2}} = \sqrt{(2.59)^2 + (0.513)^2} = \boxed{2.64 m/s} \qquad \theta = \tan^{-1}\frac{v_{1fy}}{v_{1fx}} = \tan^{-1}\frac{(0.513)}{(2.59)} = \boxed{11.2^0}$$

What amount of kinetic energy is lost? Remember energy is a directionless scalar, so x and y are meaningless.

$$(K_{1i} + K_{2i}) - K_{lost} = (K_{1f} + K_{2f})$$

$$\left(\frac{1}{2} m v_{1i}^{\ 2} + \frac{1}{2} m v_{2i}^{\ 2}\right) - K_{lost} = \left(\frac{1}{2} m v_{1f}^{\ 2} + \frac{1}{2} m v_{2f}^{\ 2}\right)$$

$$\left(\frac{1}{2}(2)(4)^2 + \frac{1}{2}(1)(0)^2\right) - K_{lost} = \left(\frac{1}{2}(2)(2.64)^2 + \frac{1}{2}(1)(3)^2\right) \qquad \boxed{K_{lost} = 0.0304 J}$$

Example 8-3: Ballistic Pendulum

The ballistic pendulum is used to determine projectile speed. The sequence of events is as follows. First a projectile, like a bullet (b), is fired into a block (B). This collision is perfectly inelastic, so

$\boxed{m_b v_{bi} + m_B v_{Bi} = (m_b + m_B) v_f}$ is used to solve for the v_f of the bullet block combination. v_f for this first phase becomes the v_0 for the second phase. In the second phase the bullet block combination swings as a pendulum to a new height, as shown in the diagram to the right. Conservation of energy applies. $\boxed{\frac{1}{2} m v_0^{\ 2} = mgh}$ is used to determine the height of the swing.

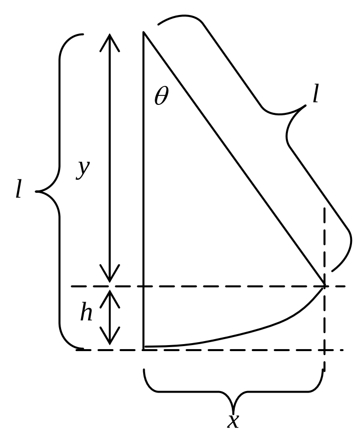


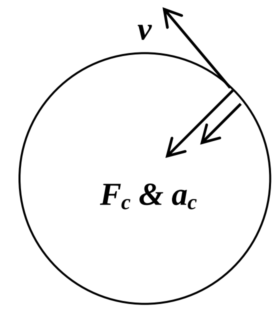
Fig 8.3

But, the point is to find the velocity of the bullet, so this problem is actually done backwards. The length of the rope holding the pendulum is known (l) and the distance the pendulum moves in the x direction (x) is measured (or the angle of swing, θ, is measured). You must use the geometry of a pendulum swing diagramed in Fig 8.2 to find the height (h) that the pendulum rises to $\boxed{y = \sqrt{l^2 - x^2}}$ and $\boxed{l = y + h}$, so $\boxed{h = l - y}$. Plug this final h into $\boxed{\frac{1}{2} m v_0^{\ 2} = mgh}$ and solve for the initial v_0 of the energy phase. Then recognize that this is the same as v_f for the collision $\boxed{m_b v_{bi} + m_B v_{Bi} = (m_b + m_B) v_f}$ Solve for v_b.

1–9 Uniform Circular Motion and Gravity

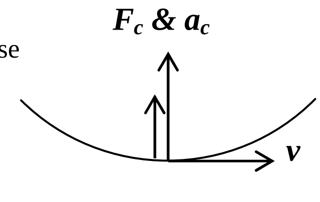
Frequency: How often a repeating event happens. Measured in revolutions per second.

Period: The time for one revolution. $\boxed{T = \frac{1}{f}}$ Time is in the numerator.



Velocity: Direction and thus velocity are continuously changing in circular motion. The magnitude of velocity and speed are not. You can measure an instantaneous velocity, which is tangential to the curve. Tangential Velocity or speed $\boxed{v = \frac{2\pi r}{T}}$

Centripetal Acceleration: Inertia would make a mass leave the circle following the tangential velocity. Instead the direction of the mass is being changed toward the center. In other words the mass is accelerated toward the center. Centripetal means center seeking. $\boxed{a_c = \frac{v^2}{r}}$



Centripetal Force: If an object is changing direction (accelerating) it must be doing so because a force is acting. Remember objects follow inertia (in this case the tangential velocity) unless acted upon by an external force. If the object is changing direction to the center of the circle it must be forced that way. $\boxed{F_c = ma_c}$ $\boxed{F_c = m\frac{v^2}{r}}$

Problem Solving Strategy

1. **As always, ask what the object is doing.** If it is moving in a circle, or even part of a circle, shown above right.
2. **Draw a FBD.** Remember ***F_c* is the sum of force** for circular motion. ***The sum of force is not shown in the FBD***.
3. **Set the direction of motion as positive.** Toward the center is positive, since this is the desired outcome.
4. **Identify the sum of force equation.** In circular motion F_c is the sum of force. F_c can be any of the previous forces.
5. **Substitute the relevant force equations and solve.**

Example 9-1: Vertical Circular Motion

A ball at the end of a string is swung in a vertical circle. Any forces pointing to the center are positive, while force vectors pointing away from the center are negative. Sum the forces. In circular motion F_c is the sum of force. **Find the tension in the string when the ball is at the top and at the bottom.**

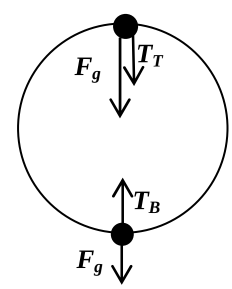


Fig 9.1

$$F_c = F_g + T_T \qquad F_c = -F_g + T_B$$

$$T_T = F_c - F_g \qquad T_B = F_c + F_g$$

$$\boxed{T_T = m\frac{v^2}{r} - mg} \qquad \boxed{T_B = m\frac{v^2}{r} + mg}$$

Example 9-2: Horizontal Circular Motion

A penny on a circular disk rotating horizontally (or a car turning a corner). Something must be keeping it going in a circle. Friction keeps it in place. If friction let go the penny would move due to inertia in a direction tangent to the disk. Force centripetal is the sum of forces for circular motion. **Find a formula for the maximum velocity if the coefficient of friction is known.**

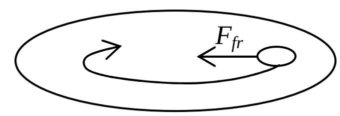


Fig 9.2

$$F_c = F_{fr} \qquad \cancel{m}\frac{v^2}{r} = \mu \cancel{m} g \qquad \boxed{v = \sqrt{\mu g r}}$$

Example 9-3: Top of a Loop and Apparent Weightlessness

Apparently weightless means that you are in freefall. The only force acting on you is F_g. To feel weightless at the top of the loop the roller coaster car can have no F_N (no pressure from the track). So for an instant at the top the car is not touching the track. **What at the top of the loop makes this possible?**

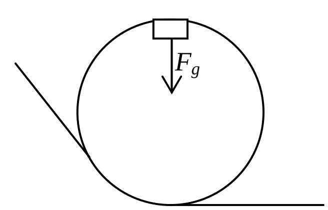


Fig 9.3

$$F_c = F_g \qquad \cancel{m}\frac{v^2}{r} = \cancel{m}g$$

Example 9-4: Conical Pendulum

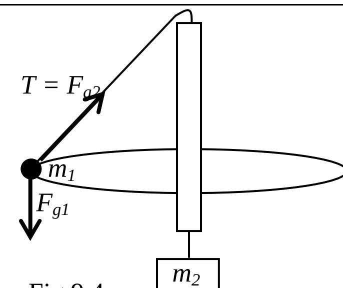


Fig 9.4a

m_1 is suspended by a string that passes through a tube. At the other end of the tube m_2 is hanging from the same string. m_1 is spun at a velocity that keeps m_2 stationary.

Solve for the force centripetal. Force centripetal is the sum of force that points to the center of the circular motion. The two acting forces on m_1 are causing the circular motion, and they must sum together as force centripetal. If you add the two acting force vectors tip to tail they form a force vector triangle, shown in Fig 9.4b.

$$T^2 = F_c^{\,2} + F_{g1}^{\,2} \qquad \boxed{F_c = \sqrt{T^2 - F_{g1}^{\,2}}}$$

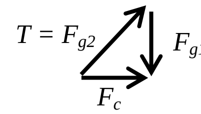
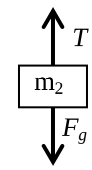


Fig 9.4b

Solve for the tension in the rope. Both masses hang from the rope, so either one can be used. Pick the easiest, in this case the vertically hanging mass. It's FBD is shown in Fig 9.4c. $\sum F = T - F_{g2}$ $\quad 0 = T - F_{g2}$ $\quad \boxed{T = F_{g2}}$

Fig 9.4c


$$F_c = \sqrt{T^2 - F_{g1}^{\,2}} \qquad m\frac{v^2}{r} = \sqrt{F_{g2}^{\,2} - F_{g1}^{\,2}} \qquad \boxed{v = \sqrt{\frac{r\sqrt{F_{g2}^{\,2} - F_{g1}^{\,2}}}{m}}}$$

Example 9-5: Gravitron

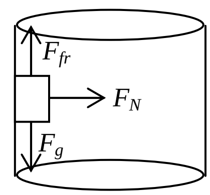


Fig 9.5

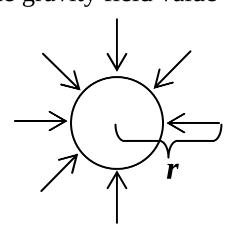
This is the ride at amusement parks where it spins and the floor drops down, leaving the occupants stuck to the wall.

Solve for the tangential velocity. You feel pressed against the wall because the wall exerts a normal force toward the center. In other words the normal force is force centripetal, $F_c = F_N$. In the vertical dimension you are prevented from sliding down the wall by an upward and equal friction force, $F_{fr} = F_g$. Friction depends on force normal. $F_{fr} = \mu F_N$. Put all these equations together, and substituting for F_c and F_g.

$$F_c = F_N \qquad F_c = \frac{F_{fr}}{\mu} \qquad F_c = \frac{F_g}{\mu} \qquad m\frac{v^2}{r} = \frac{F_g}{\mu} \qquad \cancel{m}\frac{v^2}{r} = \frac{\cancel{m}g}{\mu} \qquad \boxed{v = \sqrt{\frac{rg}{\mu}}}$$

Gravity: One of the fundamental forces. This force is a field force, and the field is g, the acceleration of gravity. Every mass in the universe generates a gravity field. The gravity field is directed toward the center of mass. While the nature of the force is not understood the mathematics are detailed in Newton's Law of Universal Gravitation $\boxed{F_g = G\frac{m_1 m_2}{r^2}}$ where $G = 6.67\times10^{-11}\ N\cdot m^2/kg^2$. This equation is the force between two masses. Remember the force between objects is equal and opposite. Combine this with the equation for weight $\boxed{F_g = mg}$ to get $mg = G\frac{m_1 m_2}{r^2}$, which simplifies as $\boxed{g = G\frac{m}{r^2}}$.

Each equation has its usefulness depending on the situation. The last equation is important for finding the gravity field value g around any mass at a distance r. To find the gravity at a point in space near Earth, use the mass of earth (which creates the gravity) and the distance from Earth's center. r is not a radius, but is the distance measured from center of a mass. r is used since gravity radiates in rays from the center of mass in a spoke like manner. Viewed in this way every distance in gravity is a radius. If you calculate g at a point in space near a mass you also know g for all points on a sphere of that radius (equipotential, since all points have the same potential energy).



Inverse Square Law: This can be used for both formulas with r^2 in the denominator. If r doubles (x2), invert to get ½ and then square it to get ¼. Gravity is ¼ its original value so F_g is ¼ of what it was and g is ¼ of what it was. Multiply the old F_g by ¼ to get the new weight, or multiply g by ¼ to get the new acceleration of gravity.

Example 9-6: Gravity on an Unknown Planet

Mars has roughly half the radius of Earth and has one-tenth the mass.

What is the gravity on the surface of Mars? Many students want to look up the radius and mass of Mars and plug into this equation $\boxed{g = G\frac{m}{r^2}}$. But there is another way. The problem gives us the relationship to Earth for a reason. We also already know the gravity on Earth $9.8 = G\frac{m_{Earth}}{r_{Earth}^{\,2}}$. What if we just use some logic and the Inverse Square Law? The gravity on Mars is going to be Earth's gravity adjusted by pretending Earth shrinks to half its radius and one-tenth its mass.

$g_{Mars} = G\frac{(m_{Earth} \times 0.1)}{(r_{Earth} \times 0.5)^2}$ $\quad$ $g_{Mars} = G\frac{m_{Earth}}{r_{Earth}^{\,2}} \times \frac{(0.1)}{(0.5)^2}$ and we already know that $9.8 = G\frac{m_{Earth}}{r_{Earth}^{\,2}}$, so

$g_{Mars} = 9.8 \times \frac{(0.1)}{(0.5)^2}$ $\quad$ $\boxed{g_{Mars} = 3.92\, m/s^2}$. Again, just pretend Earth shrinks to become Mars. This last line is all the work you need to show.

Example 9-7: Superposition of Gravity Fields

Superposition is a term referring to the addition, or superimposing, of two or more force fields. In Fig 9.7a mass A and mass B both create gravity at all points in space to infinity. If an object is positioned at point P it will feel the gravity of both masses. The two gravities must be added together using vector addition. $m_A = 2.00\times10^{20}$ kg, $m_B = 4\times10^{20}$ kg, The masses are 2.00×10^{8} m apart. Point P is located at a point half way between the masses.

What gravity is felt at point P? First solve for the gravity of each planet, at point P, separately. Use $g = G\frac{m}{r^2}$

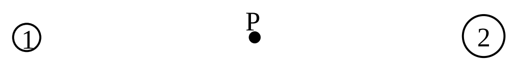


Fig 9.7a

$g = \left(6.67\times10^{-11}\right)\frac{\left(2.00\times10^{20}\right)}{\left(1.00\times10^{8}\right)^2} = \boxed{1.33\times10^{-6}\, m/s^2}$ toward m_1 (left)

$g = \left(6.67\times10^{-11}\right)\frac{\left(4.00\times10^{20}\right)}{\left(1.00\times10^{8}\right)^2} = \boxed{2.67\times10^{-6}\, m/s^2}$ toward m_2 (right)

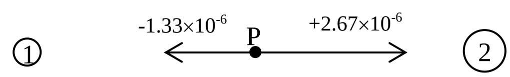


Fig 9.7b

g is a vector, and vectors have direction. Assign a positive sign to the vector pointing right and a negative sign to the vector pointing left, as shown in Fig 9.7b. Then add the two vectors together $\left(-1.33\times10^{-6}\right) + \left(+2.67\times10^{-6}\right) = \boxed{+1.33\times10^{-6}}$.

The positive answer implies that the gravity at point P is 1.33×10^{-6} m/s^2 directed toward the right.

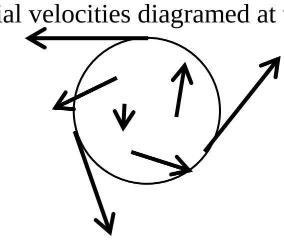
If a 100 kg mass were to be positioned at point P, what would the force of gravity be? The beauty of finding g is that you can easily apply it to any mass at that location to find the force of gravity $F_g = (100)\left(1.33\times10^{-6}\right) = \boxed{1.33\times10^{-4}\, N}$

Potential Energy Revisited:

There is another equation to find potential energy using the universal gravity constant. Use the work formula and work energy theorem, $W_g = \Delta U_g = F_g \Delta r$. Set the initial displacement as zero and it simplifies to $U_g = F_g r$. Use this with $\boxed{F_g = -G\frac{m_1 m_2}{r^2}}$ to get $\frac{U_g}{r} = -G\frac{m_1 m_2}{r^2}$. This simplifies to $\boxed{U_g = -G\frac{m_1 m_2}{r}}$.

Where did the minus sign come from? Suddenly it is added to Newton's Law of Universal Gravitation. This is the formal version of the law. It can be used with either a positive sign (simplified and common version) or a negative sign (formal version) and is situational dependent. In formal physics a point at infinity is said to have zero potential energy. Since a central point of zero potential energy cannot be located in the universe, it makes sense to pick infinity to be zero potential energy. All points in the universe are the same infinite distance from infinity. However, this means that close to Earth's surface potential energy is negative. It is common practice when viewing planets from a great distance to set infinity as $U_g = 0$, and when on a planets surface to set the lowest height as $U_g = 0$. These are just conventions used to make specific problems easier to solve. Remember, the exact energy that an object has is not really important. What matters is how much of that energy is usable to do work. And work is a change in energy. Therefore, we can really declare any point as zero energy and measure changes from that point.

1–10 Introduction to Rotation and Torque



Rotation: In rotation the entire object spins around its center of mass. Looking at the tangential velocities diagramed at the right, we see that they are all in different directions and all vary in magnitude. Points near the outer edge have to move through a larger circumference in the same period than those closer to the center. The outer edge must be moving faster to cover the longer distance in the same period or time. All of these points have one thing in common, they all travel through the same number of degree or radians during a period. Rotational velocity measured in radian per second is called **angular velocity**. However, **All the equations for an object in circular motion hold true if we are looking at a single point and only a specific point on a rotating object.** Rotating objects have **rotational inertia** and an accompanying **angular momentum**, meaning that a rotating object will continue to rotate (or not rotate) unless acted upon by an **unbalanced torque**, discussed below.

(Note: Planets and satellites follow circular motion, as they are not attached. Inner planets move faster as they are closer to the sun and must have larger tangential velocities. They also travel a shorter circumference. Thus they have shorter periods.)

Angular momentum: Depends on mass (like regular momentum) and it also depends on mass distribution. As an ice skater brings their arms closer to the body they begin to spin faster, since the mass has a shorter distance to travel.

Angular momentum is conserved. The radius gets smaller, but angular velocity increases (vice versa as the skater moves arms outward). A galaxy, solar system, star, or planet forms from a larger cloud of dust. As the cloud is pulled together by gravity its radius shrinks. So the angular velocity must increase. These objects all begin to spin faster. That is why we have day and night.

Torque: In rotation problems we look at the sum of torque (not the sum of force). But it is exactly the same methodology.

$\boxed{\tau = rF\sin\theta}$ Strongest when the force is **perpendicular** to the lever arm (since sin 90^o equals one).

Balanced Torque: The sum of torque is zero. No rotation.

Unbalance Torque: Adding all the clockwise and counterclockwise torque does not sum to zero. So there is excess torque in either the clockwise or counterclockwise direction. This will cause the object to rotate.

1. **As always, ask what the object is doing.** Is it rotating or is it standing still?
2. **Set the direction of motion as positive.** The convention when in doubt is that counterclockwise is positive. This corresponds to projectile motion where angles measured from the horizon counterclockwise were positive. But, just like in forces if you know the direction of motion call it positive. It will either rotate clockwise or counterclockwise. If you pick the wrong direction your final answer will be negative. But, the answer will be correct nonetheless. If it is not moving pick one direction to be positive, it really doesn't matter. But the other must be negative, so the opposing torques cancel.
3. **Identify the sum of torque equation.**

$\sum\tau = \sum\tau_{cw} - \sum\tau_{ccw}$ **or** $\sum\tau = \sum\tau_{ccw} - \sum\tau_{cw}$

4. **Substitute the relevant force equations and solve** (examples assume clockwise was positive direction)

Rotating: $\sum\tau = \sum(rF\sin\theta)_{cw} - \sum(rF\sin\theta)_{ccw}$

Not Rotating: $0 = \sum(rF\sin\theta)_{cw} - \sum(rF\sin\theta)_{ccw}$ $\sum(rF\sin\theta)_{cw} = \sum(rF\sin\theta)_{ccw}$

Example 10-1: Torque and a Seesaw

Three masses are positioned on a seesaw as shown in Fig 10.1. m_A = 4.0 kg, m_B = 2.0 kg, and m_3 = 3.0 kg. Distances are shown in the diagram.

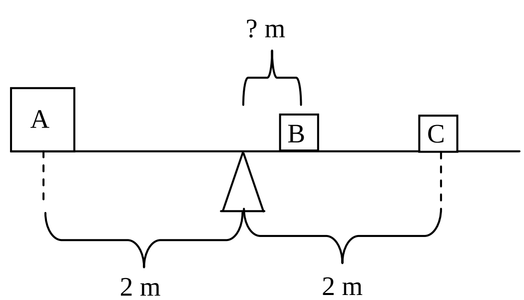


Fig 10.1

How far from the fulcrum must m_B be positioned in order for the system to balance? Keep in mind that measurements are made from the center of mass. It is as though all the mass is mathematically located at a point at the center of the object. It is not rotating, so the clockwise torques must equal the counterclockwise torques.

$$\sum\tau_{cw} = \sum\tau_{ccw}$$

$$\tau_A = \tau_B + \tau_C$$

$$r_A \cdot F_A = r_B \cdot F_B + r_C \cdot F_C$$

$$r_A \cdot m_A \cancel{g} = r_B \cdot m_B \cancel{g} + r_C \cdot m_C \cancel{g}$$

$$(2)(4) = r_B(2) + (2)(3) \qquad r_B = \boxed{1.0m}$$

1−11 Rotation Detailed, Rolling, and Angular Momentum

Note: This detail on rotation is needed for AP Physics C: physical science majors, calculus based.

Center of Mass: Objects rotate around a central axis and around a center of mass. It is therefore important to be able to locate the center of mass. The center of mass is each for shapes like squares, rectangles, circles, spheres, or equilateral triangles. It is in the middle. The following equation will find the center of mass of a system of point masses or for a system of geometric shapes (those just mentioned, that you can find the center of by inspection) $\boxed{\mathbf{r}_{cm} = \sum m\mathbf{r} / \sum m}$. Unusual shapes can be found experimentally by hanging the object from two or more positions, drawing vertical lines from the point of attachment of the string, and looking for an intersection. Or integral calculus can be used. For this course these last two methods will not be discussed.

Example 11-1: Center of Mass

Find the center of mass for the object in Fig 11.1a. It is a thin flat object composed of a rectangle (2m by 4m in length, mass 5 kg) and a square (2m long sides, mass 3 kg). Set up a coordinate axis system. For convenience place the coordinate axis at one corner of the object and divide the object into a rectangle and a square, as shown in Fig 11.1b. Find the center of mass the rectangle, relative to the coordinate axis, by inspection, $x = 1,\ y = 2$. Find the center of mass the rectangle, relative to the coordinate axis, by inspection, $x = 3,\ y = 1$. **Now you can pretend that the rectangle and square are point masses at these locations. The remainder of the problem is the method for solving for point masses in two dimensions.** You must work in each dimension separately.

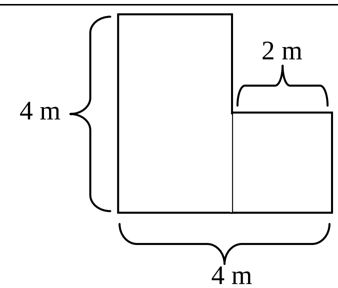


Fig 11.1a

$$x_{cm} = \frac{\sum mx}{\sum m} \qquad x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} \qquad x_{cm} = \frac{(5)(1)+(3)(3)}{(5)+(3)} = 1.75m$$

$$y_{cm} = \frac{\sum my}{\sum m} \qquad y_{cm} = \frac{m_1y_1 + m_2y_2}{m_1 + m_2} \qquad x_{cm} = \frac{(5)(2)+(3)(1)}{(5)+(3)} = 1.63m$$

The center of mass is located at $\boxed{\mathbf{r}_{cm} = 1.75\mathbf{i}m + 1.63\mathbf{j}m}$

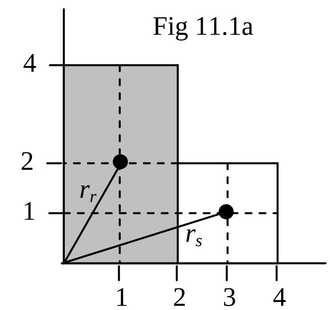


Fig 11.1b

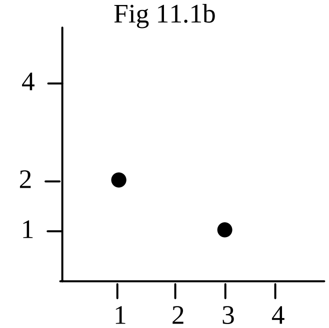


Fig 11.1c

Rotation: Since every point on a rotating object experiences a different tangential velocity displacement, velocity, and acceleration cannot be expressed in terms of meters. A particle on the outside edge of a rotating object covers a greater distance in the same time interval than a particle closer to the center. The only quantity that both points share in any given time interval is the angle through which they move, as shown to the right. In rotation we have to work in radians instead of degrees. This means that for every variable in linear (***translational***) motion there is a corresponding variable for rotation. And every equation in linear motion has a rotational counterpart. Displacement x is replaced by ***radians*** θ (radians). Velocity v is replaced by ***angular velocity*** ω (radians per second). Acceleration a is replaced by ***angular acceleration*** α (radians per second squared) The following three equations form a bridge between linear motion and rotation and should be memorized. $\boxed{x = r\theta}$ $\boxed{v = r\omega}$ $\boxed{a = r\alpha}$. The chart below, and on the following pages, compares rotation to linear motion. There is an analogous quantity and an analogous equation for rotation that parallels those learned in linear translational motion. Keep the three equations listed above in mind and become familiar with the new quantities.

	Angular	Linear
Position	$\theta = \frac{arc\ length}{r}$	x
Displacement	$\Delta\theta = \theta - \theta_0$	$\Delta x = x - x_0$
Average Speed	$\bar{\omega} = \frac{\Delta\theta}{\Delta t}$	$\bar{v} = \frac{\Delta x}{\Delta t}$ $\quad \bar{v} = \frac{v_0 + v}{2}$
Instantaneous Speed	$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$ $\quad \omega = \frac{d\theta}{dt}$	$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$ $\quad v = \frac{dx}{dt}$ **Slope of displacement - time graph**
Average Acceleration	$\bar{\alpha} = \frac{\Delta\omega}{\Delta t}$	$\bar{a} = \frac{\Delta v}{\Delta t}$
Instantaneous Acceleration	**Tangential Acceleration** $\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t}$ $\quad \alpha = \frac{d\omega}{dt}$	$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$ $\quad a = \frac{dv}{dt}$ **Slope of velocity - time graph**
Kinematic Equations	$\boxed{\omega = \omega_0 + \alpha t}$ $\boxed{\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2}$ $\omega^2 = \omega_0{}^2 + 2\alpha\left(\theta - \theta_0\right)$	$\boxed{v = v_0 + at}$ $\boxed{x = x_0 + v_0 t + \frac{1}{2}at^2}$ $\boxed{v^2 = v_0{}^2 + 2a\left(x - x_0\right)}$
Tangential Speed	$\boxed{v = r\omega}$ $\quad \omega = \frac{2\pi}{T}$	$v = \frac{2\pi r}{T}$
Centripetal Acceleration	**Radial Acceleration** $\boxed{a_c = \frac{v^2}{r} = \omega^2 r}$ Radial Acceleration is the acceleration directed along a radial (spoke) line. It is directed toward the center.	$a_c = \frac{v^2}{r}$
Inertia	**Moment of Inertia:** Depends on mass and distribution and thus varies for each object $\boxed{I = \int r^2 dm = \sum mr^2}$ Since these vary from object to object they are usually given. The three shown here are commonly used. The first one is the common shape for pulley, which are the most used. Cylinder: $\boxed{I = \frac{1}{2}MR^2}$ Cylindrical hoop: $I = MR^2$ Sphere: $I = \frac{2}{5}MR^2$	m
Force and Torque	**Torque:** Unbalance torques cause rotation. $\boxed{\tau = \mathbf{r} \times \mathbf{F}}$ $\boxed{\sum \tau = \tau_{net} = I\alpha}$	**Force:** Unbalanced forces cause translation. $\mathbf{F}$ $\boxed{\sum \mathbf{F} = \mathbf{F}_{net} = m\mathbf{a}}$
Kinetic Energy	$\boxed{K = \frac{1}{2}I\omega^2}$	$\boxed{K = \frac{1}{2}mv^2}$

	Angular	Linear
Work	$W = \int \tau d\theta$ $W = \frac{1}{2}I\omega^2 - \frac{1}{2}I\omega_0{}^2$	$\boxed{W = \int \mathbf{F} \cdot d\mathbf{r}}$ **Area under force – distance curve** $W = \frac{1}{2}mv^2 - \frac{1}{2}mv_0{}^2$
Power	$\boxed{P = \frac{dW}{dt}}$ $P = \tau\omega$	$\boxed{P = \frac{dW}{dt}}$ $\boxed{P = Fv}$

Vector Product and Torque: Torque is a cross product of vectors. The **magnitude of a cross product is the area of the parallelogram formed by the contributing vectors**. The **direction of a cross product vector is determined by using the right hand rule**. So the direction of torque is out of the page for counterclockwise rotation, and into the page for clockwise rotations.

Translation vs. Rotation: Hit an object with a force directed into or out of the center of mass and it will translate (linear motion). Hit an object with a force perpendicular to a radial line extending from the center of mass and at the very edge of the object, and the object will rotate. Hit and object with a force between the center of mass and the edge and it will translate and rotate.

(Note: Planets and satellites follow circular motion, as they are not attached. Inner planets move faster as they are closer to the sun and must have larger tangential velocities. They also travel a shorter circumference. Thus they have shorter periods.)

Angular momentum: Masses that experience linear motion (translation) have velocity and thus have linear momentum. Rotating masses have angular velocity and thus have angular momentum. While linear momentum depends on mass and velocity, angular momentum depends on mass, mass distribution, and angular velocity. Think about it. In rotating objects the points of mass farther from the center are moving faster and thus have higher instantaneous momentum values than those closer to the center. Lots of mass, far from the center of mass, means higher angular momentum than the same mass, near the center of mass.

Angular momentum is conserved. The radius gets smaller, but angular velocity increases (vice versa as the skater moves arms outward). A galaxy, solar system, star, or planet forms from a larger cloud of dust. As the cloud is pulled together by gravity its radius shrinks. So the angular velocity must increase. These objects all begin to spin faster. That is why we have day and night.

	Angular	Linear
Momentum	$\boxed{\mathbf{L} = \mathbf{r} \times \mathbf{p} = I\boldsymbol{\omega}}$	$\boxed{p = mv}$
Conservation of Momentum	$\mathbf{L}_i = \mathbf{L}_f$ $I\boldsymbol{\omega}_i = I\boldsymbol{\omega}_f$	$p_i = p_f$ $mv_i = mv_f$

Example 11-2: Compound Bodies and Pulleys with Mass

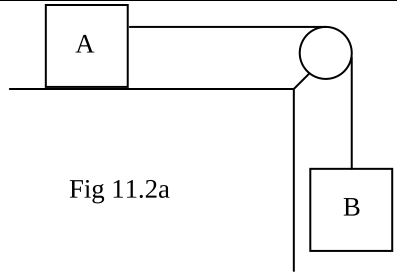


Fig 11.2a

A compound body consisting of, m_A = 6.0 kg, m_B = 8.0 kg, M_{pulley} = 1 kg, R_{pulley} = 0.10 m, is shown in Fig. 11.1a.

What is the acceleration of the system?

There are three masses, so there are three FBD's, shown in Fig 11.2b. Make note of the interesting new FBD for a pulley. Gravity acts through the center and down, as usual. The normal force is created by the support pushing the pulley away from the table, and it follows the direction of the support through the center of the pulley. ***The tensions are tangent to the pulley. These tensions are a distance R (radius of pulley) from the center and they are perpendicular to the R. This provides the torque that rotates the pulley. Also note that there are two tensions. When we work with real pulleys that have mass the rope connecting the masses has different tensions in every separate segment.***

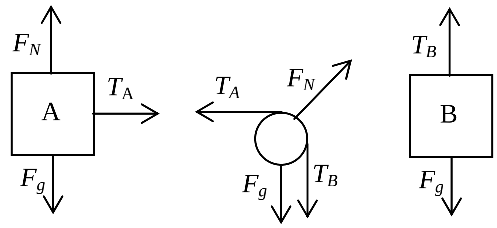


Fig 11.2b

Set up sum of force and sum of torque equations for relevant masses. As before use the direction of motion to assign positives and negatives. The direction of motion is to the right, clockwise, and then down.

$$\sum F_A = T_A \qquad \sum \tau = \tau_{cw} - \tau_{ccw} \qquad \sum F_B = F_g - T_B$$

$$m_A a = T_A \qquad I\alpha = R\cdot T_B - R\cdot T_A \qquad m_B a = m_B g - T_B$$

$$T_A = m_A a \qquad I\frac{a}{R} = R\cdot T_B - R\cdot T_A \qquad T_B = m_B g - m_B a$$

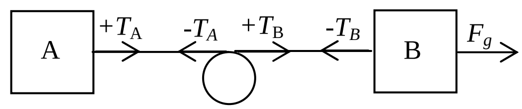


Fig 11.2c

Combine the three equations above to get

$I\dfrac{a}{R} = R\left(m_B g - m_B a\right) - R\left(m_A a\right)$ Substitute in the moment of inertia of a cylindrical disk (pulley) $I = \dfrac{1}{2}MR^2$

$\left(\dfrac{1}{2}MR^2\right)\dfrac{a}{R} = R\left(m_B g - m_B a\right) - R\left(m_A a\right)$ Cancel out the pulleys radius, group all expression with a, and simplify.

$$\frac{1}{2}Ma = m_B g - m_B a - m_A a \qquad m_A a + m_B a + \frac{1}{2}Ma = m_B g \qquad \boxed{a = \frac{m_B g}{\left(m_A + m_B + \frac{1}{2}M\right)}}$$

This looks familiar. If we did the problem the old way, with a massless pulley, we would look at it as linear, like Fig 11.2c

It would be $\sum F_{total} = F_{gB}$ $\qquad \left(m_A + m_B\right)a = m_B g \qquad a = \dfrac{m_B g}{\left(m_A + m_B\right)}$

This is identical except for the expression for half the pulley's mass. Can we just do all pulley problems the old way and just add a $\frac{1}{2}M$ to all the regular masses in the denominator. It seems to work, but you might loose points for not showing work.

And the $\frac{1}{2}M$ only works with pulleys that have a moment of inertia of $I = \frac{1}{2}MR^2$. If it were a spherical pulley, would we add $\frac{2}{5}M$ to the denominator, since its moment of inertia is $I = \frac{2}{5}MR^2$. Verify it on your own and see.

Fig 11.2c shows the problem sketched linear. Any forces that are perpendicular to the direction of motion were removed from this sketch. Vectors pointing in the direction of motion are noted with positive signs and those opposing motion are negative. It is apparent that tension cancels as before. However, unlike previous work in forces the pulley is not erased as it now has mass. It must be accounted for. Is there a shortcut method using this linear sketch that would show adequate supporting work?

Example 11-3: Rolling Down an Incline

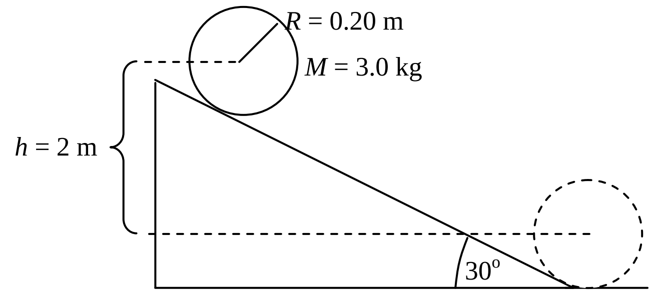


Fig 11.3a

A spherical mass rolls 2 m down an incline shown in Fig 11.3a. The FBD for the sphere is shown in Fig 11.3b. In figure 11.3c the gravity and normal force vectors have been summed and the component of force down the slope is shown. $F_g\sin\theta$ pulls the sphere down the incline in the usual manner. However, the friction force vector is a distance R from the center of the sphere. These two vectors are perpendicular. This creates an unbalanced torque on the sphere, which causes it to rotate. The combined motion of rotating and moving down the slope is rolling.

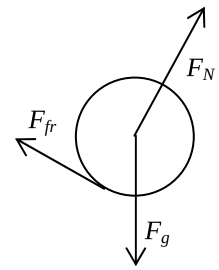


Fig 11.3b

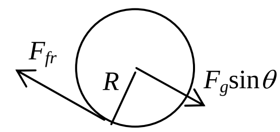


Fig 11.3c

What is the kinetic energy of the sphere at the bottom of the incline? The sphere is translating and rotating at the same time. The total kinetic energy is the addition of the translational and rotational kinetic energies.

$$K_{total} = K_{translation} + K_{rotation}$$

$$K_{total} = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

It is also equal to the potential energy that converted in kinetic energy. Given the quantities in the problem, this is easiest to solve

$$K_{total} = mgh \qquad K_{total} = (3.0)(9.8)(2.0) = \boxed{58.8J}.$$

How fast is the sphere going at the bottom of the incline?

Now the first kinetic energy equation has relevance, $K_{total} = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$. Combine this with the moment of inertia equation of a sphere $\frac{2}{5}MR^2$, and the equation $\omega = \frac{v}{R}$ which converts angular values into linear values.

$$K_{total} = \frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{v}{R}\right)^2 \qquad K_{total} = \frac{1}{2}Mv^2 + \frac{2}{10}Mv^2 \qquad K_{total} = \frac{7}{10}Mv^2$$

Rearrange for velocity, plug in values, and solve.

$$v = \sqrt{\frac{10}{7}\frac{K_{total}}{M}} \qquad v = \sqrt{\frac{10}{7}\left(\frac{58.8}{3.0}\right)} = \boxed{5.29\,m/s}$$

Another expression can be derive here also $v = \sqrt{\frac{10}{7}gh}$

What is the linear acceleration of the sphere down the inline?

The length down the ramp is $hyp = \frac{h}{\sin\theta}$

$$v^2 = v_0{}^2 + 2a(x - x_0)$$

$$\left(\sqrt{\frac{10}{7}\frac{K_{total}}{M}}\right)^2 = (0)^2 + 2a((x) - 0)$$

$$\left(\sqrt{\frac{10}{7}\frac{Mgh}{M}}\right)^2 = (0)^2 + 2a\left(\frac{h}{\sin\theta}\right)$$

$$a = \frac{5}{7}g\sin\theta \qquad a = \frac{5}{7}(9.8)\sin 30^o = \boxed{3.5\,m/s^2}$$

Unit 2 Fluids and Thermal Physics

2–12 Fluid Mechanics

Solids: ***Condensed matter*** with definite volume and shape. High intermolecular forces create rigid structure. Force can deform it.

Liquids: ***Condensed matter and fluid*** with fairly definite volume, but takes the shape of its container. Loose intermolecular forces.

Gases: ***Fluid only***. Easily compressed, taking shape & volume of its container. Particle speed interferes with intermolecular force.

Fluids: ***Substance that can flow*** (gases and Liquids). Outside forces cause, unconfined, fluids to flow.

Pressure: ***Force applied over an area***. $\boxed{p = \frac{F}{A}}$ F is perpendicular to the surface (normal). Pressure is a scalar quantity.

Measured in Pascals. 1 Pa = 1 N/m^2. 1 atm = 101325 Pa, which simplifies to $\boxed{1atm = 1\times10^5\,Pa}$

Density: ***The amount of matter occupying a certain amount of space***. $\boxed{\rho = \frac{m}{V}}$ Measured in kg/m^3 (in chem. g/cm^3)

The density of water is easy to memorize. Water is the most common substance on earth, so chemists set its value at 1 g/cm^3.

Water is the reference standard by which all other densities are compared. But, physics operates on a larger scale with different units. $\rho_{H_2O} = 1.00\,g/cm^3 = \boxed{1000\,kg/m^3}$

Pressure and Depth: ***Pressure increases with depth.*** A column of fluid (air we breath or water in oceans) has weight and puts a force on the fluid below it, Fig 12.1. The deeper one is (on the surface of earth for air, or at the bottom of the ocean for water) the greater the force and pressure. The equation for pressure at depth is derived by combining the following four equations.

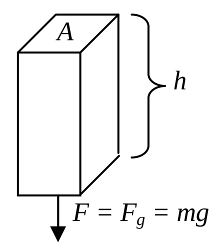


Fig 12.1

$$p = \frac{F}{A}, \quad F_{due\ to\ fluid\ above} = mg, \quad \rho = \frac{m}{V}, \text{ and } \quad V = hA \quad \longrightarrow \quad p = \rho g h$$

which becomes $\boxed{p = p_0 + \rho g h}$ p_0 is the pressure on the surface. This could be caused by a piston in a closed tube, or by the atmospheric pressure for a fluid whose surface is exposed. Example: a scuba diver, at a depth of 80 ft in saltwater, experiences a pressure of

$p = p_0 + \rho g h = (1\times10^5) + (1.03\times10^3)(9.8)(24.4) = \boxed{3.46\times10^5\,Pa} = 3.46atm$ (3.5 × atmospheric pressure at sea level). How does this translate to force? Calculate the force on 1 square meter $p = F/A$, so

$F = pA = (3.46\times10^5\,Pa)(1m^2) = \boxed{34600N}$

Pascal's Principle: ***Pressure applied to an enclosed fluid is transmitted, undiminished, to every point in the fluid and to the walls of the container.*** This explains the workings of hydraulic lifts, jacks, and brakes. The pressure put in (input) at one end is equal to the pressure put out (output) at the other.

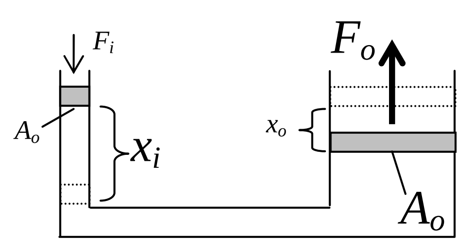


Fig 12.2

$p_{input} = p_{output}$. Substitute $p = \frac{F}{A}$ on each side to get $\boxed{\frac{F_i}{A_i} = \frac{F_o}{A_o}}$.

A small input force on a small area creates a large output force on a large area. Work done pushing the piston can also be analyzed.

$W_i = W_o$ which is $F_i x_i = F_o x_o$ and becomes $\boxed{F_o = F_i \frac{x_i}{x_o}}$

The input distance must be large to get a large output force, F_o.

Buoyancy: Things float because they are buoyed upward, which requires an upward net force on the object. It is caused by the difference in pressure $\left(\Delta p = p_{bottom} - p_{top}\right)$ between the bottom of the object and the top of the object.

$p = \frac{F}{A}$ combined with $\Delta p = \rho g h_{bottom} - \rho g h_{top} = \rho g\left(h_{bottom} - h_{top}\right)$ results in $\frac{F}{A} = \rho g\left(h_{bottom} - h_{top}\right)$ which is

$F = \rho g\left(h_{bottom} - h_{top}\right)A$ or $\boxed{F_{buoy} = \rho V g}$ Remember this is the ***density of the fluid*** and the ***volume of the fluid displaced***. This is the equation for the weight of the fluid displaced. $F_{buoy} = \rho V g = \frac{m}{V} V g = \boxed{mg}$ of the fluid.

Archimedes' Principle: ***<u>A body immersed wholly or partially in a fluid is buoyed up by a force equal in magnitude to the weight of the volume of fluid it displaces.</u>***

Buoyancy and Density: Density tells whether it will sink or float. Fig 12.3 diagrams several masses in a fluid.

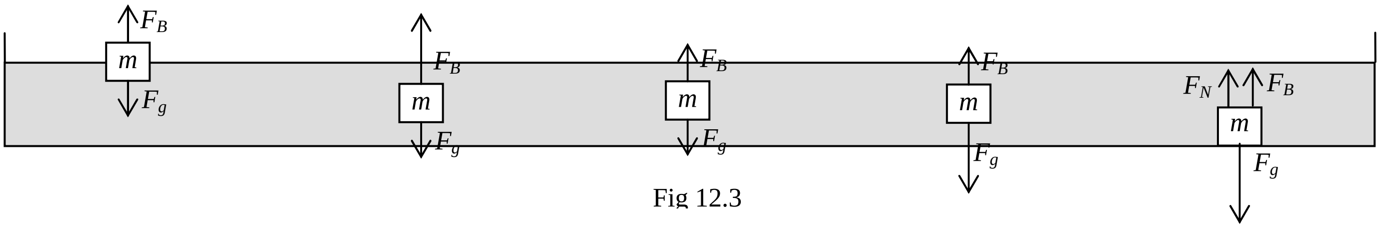


Fig 12.3

Floats out of water	Rising to Surface	Neutral Buoyancy	Sinking to Bottom	Resting on Bottom
$F_B = F_g$	$F_B > F_g$	$F_B = F_g$	$F_B < F_g$	$F_B + F_N = F_g$
$\rho_{obj} < \rho_{Fluid}$	$\rho_{obj} < \rho_{Fluid}$	$\rho_{obj} = \rho_{Fluid}$	$\rho_{obj} > \rho_{Fluid}$	$\rho_{obj} > \rho_{Fluid}$
$m_{obj} = m_{Fluid\ Displaced}$	$m_{obj} < m_{Fluid\ Displaced}$	$m_{obj} = m_{Fluid\ Displaced}$	$m_{obj} > m_{Fluid\ Displaced}$	$m_{obj} > m_{Fluid\ Displaced}$
$V_{obj} > V_{Fluid\ Displaced}$	$V_{obj} = V_{Fluid\ Displaced}$	$V_{obj} = V_{Fluid\ Displaced}$	$V_{obj} = V_{Fluid\ Displaced}$	$V_{obj} = V_{Fluid\ Displaced}$

Specific Gravity: ratio of the density of a substance (ρ_s) to the density of water (ρ_w). $\boxed{sp.gr. = \rho_s / \rho_w}$

Example 1-1: Buoyancy

A 2.0 kg cube, that is 10.0 cm in length on each side, is suspended by both a gray fluid and by a spring as shown in Fig 12.4a. The spring constant is 100 N/m and the spring is stretched 0.15 m.

What is the density of the fluid? This is a balanced force problem. The object is not moving. Use the FBD in Fig 12.4b to sum the forces.

$\sum F = F_S + F_B - F_g \qquad \sum F = 0$

$F_S + F_B = F_g$ Upward force spring and force buoyancy equals the downward force gravity.

$kx + \rho V g = mg$

$\rho = \frac{mg - kx}{Vg} \qquad \rho = \frac{(2.0)(9.8) - (100)(0.15)}{(0.1)^3(9.8)} = \boxed{469\frac{kg}{m^3}}$

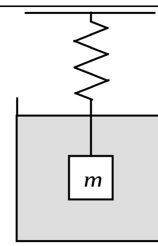


Fig 12.4a

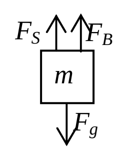


Fig 12.4b

Fluid Dynamics: The overall flow of a fluid can be described, but the motion of the individual molecules or atoms that compose the fluid cannot. We use a theoretical model, known as an ***<u>ideal fluid</u>***. Ideal fluids have four characteristics.

Steady Flow: ***<u>Smooth flow</u>***, where all the particles of a fluid have the same velocity as they pass a given point. The particle path is depicted by streamlines that never cross. This requires low velocities, since high flow rate results in eddies, particularly near boundaries, which give rise to turbulence. Relative velocity is shown by drawing faster flowing fluid streamlines closer together.

Irrotational Flow: No net angular velocities in any portion of the fluid. i.e., there is no possibility for eddies and turbulence.

Nonviscous Flow: A nonviscous fluid flows freely without energy loss. Viscosity is a fluid's internal friction, and is neglected.

Incompressible Flow: Fluid's density is constant. Gases are not incompressible, but sometimes they approximate incompressibility.

Equation of Continuity: ***If there is no loss of fluid in a closed tube the mass entering must equal the mass leaving the tube.*** In Fig 12.5 a fluid flows through a tube that is narrow at first and then widens. The volumes of the narrow and wide portions are:

$$V_1 = A_1\Delta x_1 \qquad V_2 = A_2\Delta x_2$$

Substitute $\Delta x = v\Delta t$: $\quad V_1 = A_1 v_1 \Delta t \qquad V_2 = A_2 v_2 \Delta t$.

Volume relates to mass.

$\rho = \dfrac{m}{V}$ so $m = \rho V$

Substituting $\Delta m = \rho \Delta V$: $\quad \Delta m_1 = \rho_1 A_1 v_1 \Delta t \qquad \Delta m_2 = \rho_2 A_2 v_2 \Delta t$

But, the two shaded sections of pipe were picked because they have equal volumes, one long and thin, the other short and wide. So if the volumes are the same, then the mass of fluid in the first section equals the mass of fluid in the second section.

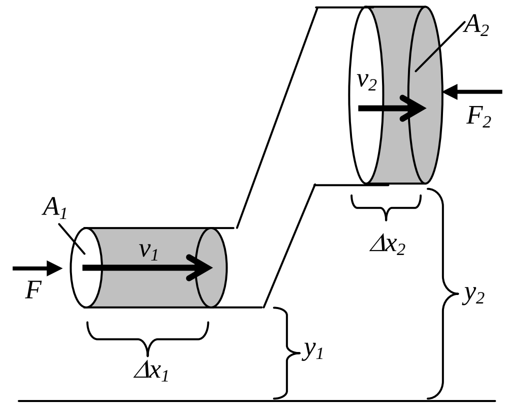


Fig 12.5

$\Delta m_1 = \Delta m_2$ and thus $\rho_1 A_1 v_1 \Delta t = \rho_2 A_2 v_2 \Delta t$. Time cancels, as does density. In ideal fluids density is assumed to be uniform.

Flow Rate Equation: $\boxed{A_1 v_1 = A_2 v_2}$ ***Flow velocity is greater where the cross sectional area is smaller.*** $v_2 = \dfrac{A_1}{A_2} v_1$

Now Analyze the Work done by external forces at ends of the tube in the figure above right.

F_1 is positive (matches motion), while F_2 is negative (opposite motion). $\quad W = F_1 \Delta x_1 - F_2 \Delta x_2$

Substitute $p = \dfrac{F}{A}$ written as $F = pA$, along with $\Delta x = v\Delta t$ $\quad W = p_1 A_1 v_1 \Delta t - p_2 A_2 v_2 \Delta t$

Using the Flow Rate Equation just derived, $A_1 v_1 = A_2 v_2$ and we simplify to $\quad W = A_1 v_1 \Delta t \left(p_1 - p_2\right)$

Then use the mass relationship from above, $\Delta m_1 = \rho_1 A_1 v_1 \Delta t$ $\quad W = \dfrac{\Delta m}{\rho}\left(p_1 - p_2\right)$

Work is a change in energy $W = \Delta K + \Delta U$ $\quad \dfrac{\Delta m}{\rho}\left(p_1 - p_2\right) = \Delta K + \Delta U$

Substitute the formulas for ΔK and ΔU

$$\frac{\Delta \cancel{m}}{\rho} p_1 - \frac{\Delta \cancel{m}}{\rho} p_2 = \frac{1}{2}\Delta \cancel{m} v_2^{\,2} - \frac{1}{2}\Delta \cancel{m} v_1^{\,2} + \Delta \cancel{m} g y_2 - \Delta \cancel{m} g y_1$$

Cancel mass and multiply all by density $\quad p_1 - p_2 = \dfrac{1}{2}\rho v_2^{\,2} - \dfrac{1}{2}\rho v_1^{\,2} + \rho g y_2 - \rho g y_1$

Rearrange separating 1's and 2's $\quad p_1 + \rho g y_1 + \dfrac{1}{2}\rho v_1^{\,2} = p_2 + \rho g y_2 + \dfrac{1}{2}\rho v_2^{\,2}$

Bernoulli's Equation: $\quad \boxed{p + \rho g y + \dfrac{1}{2}\rho v^2 = constant}$

Applying Bernoulli's Equation to various situations.

- Horizontal flow $\left(y_1 = y_2\right)$: $\boxed{p + \dfrac{1}{2}\rho v^2 = constant}$ Indicates that pressure decreases if fluids speed increases or vice versa.
- Looking at both Bernoulli's and the Continuity Equation: Reduce pipe cross section, then velocity goes up, and pressure down.
- Partially responsible for lift: Air over the curved surface of a wing goes faster, with lower pressure. So the slow high pressure air on the bottom of the wing pushes upward.
- Fluids at rest $\left(v_2 = v_1 = 0\right)$: $\boxed{p_2 - p_1 = \rho g\left(y_1 - y_2\right)}$ which is the pressure depth relationship.

Example 1-2: Buoyancy

A tube, shown in Fig 12.6a, that has an opening at the top and a hole in the side is continuously filled with water. It is filled so that the water level remains constant, even though water is leaking from the hole in the side.

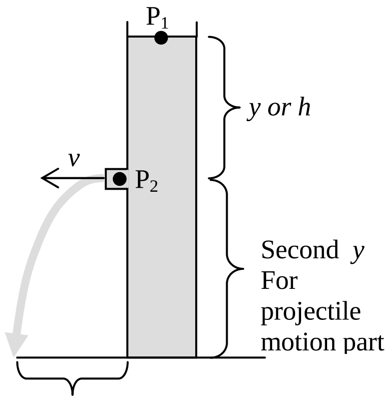


Fig 12.6a

What is the velocity of the water coming out of the hole in the side?

Use Bernoulli's Law $\boxed{p+\rho gy+\frac{1}{2}\rho v^2=constant}$

Pick two convenient locations: The surface of the water (point 1) and the hole in the side (point 2). The fluid flows from the surface $\boxed{p_1+\rho gy_1+\frac{1}{2}\rho v^2{}_1=constant}$ to the hole $p_2+\rho gy_2+\frac{1}{2}\rho v^2{}_2=constant$.

It is the same fluid, so the constant at both points must be the same. Therefore,

$$p_1+\rho gy_1+\frac{1}{2}\rho v^2{}_1=p_2+\rho gy_2+\frac{1}{2}\rho v^2{}_2$$

The tube is open to the atmosphere at both ends, so $p_1=p_2$

$$\cancel{\rho}gy_1+\frac{1}{2}\cancel{\rho}v^2{}_1=\cancel{\rho}gy_2+\frac{1}{2}\cancel{\rho}v^2{}_2$$

Since the level of the water is maintained at a constant point the velocity at point 1 is zero.

$gy_1=gy_2+\frac{1}{2}v^2{}_2$ Which simplifies to $v=\sqrt{2g\left(y_1-y_2\right)}$ or $\boxed{v=\sqrt{2g\Delta y}}$

What if we look at this differently?

Suppose we follow a single water molecule from point 1 to point 2. Pretend it follows the path (in an energy problem path doesn't matter, so any path will give the same answer) diagramed in Fig. 12.6b. What would its velocity be? Height is turning into velocity. $\cancel{m}gh=\frac{1}{2}\cancel{m}v^2$

$v=\sqrt{2gh}$ or $\boxed{v=\sqrt{2g\Delta h}}$. This looks familiar! Notice that mass cancelled, so it does not matter if it is one molecule or a stream of molecules. Funny how Physics ties together and different approaches to problems can yield the same answers.

Notice that the water leaving the opening is a horizontally launched projectile.

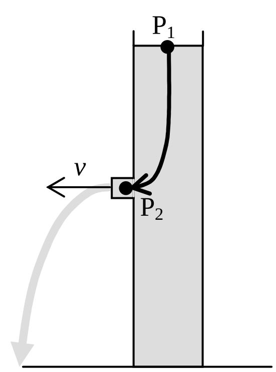


Fig 12.6b

$\boxed{y=\frac{1}{2}gt^2}$ and $\boxed{x=v_{0x}t}$ Just be careful this y is different from the y used in Bernoulli's equation. Note the labels on the right side of Fig 12.6a

2–13 Thermal Physics

Temperature: Average KE of the particles. Depends on average **speed only** of particles (atoms or molecules)
A bucket of water at 50^o has the **same temperature** as a cup of water at 50^o

Thermal Energy: Average KE and the mass of the particles Depends on **speed and the mass** of the particles.

A bucket at 50^o has **more thermal energy** than a cup at 100^o. Bucket of $50 bills has more money than a cup of $100's. While the particles are going faster in the cup, there are so many more in the bucket.

Heat: Transfer of energy between objects that have different temperatures. The direction of heat flow depends on temperature. From hot objects (more energy) to cold objects (less energy).

Thermal Expansion: As objects are heated the molecules move faster and they expand. This is why the liquid in a thermometer rises when heated. It is how a thermostat works. It is why there are gaps in sidewalks, bridges, & railroad tracks. It is why they don't fill bottles to the top, they might explode if the liquid expands to much. The change in length is $\boxed{\Delta\ell = \ell_0\alpha\Delta T}$. Original length ℓ_0 is multiplied by α, the coefficient of linear expansion, and the change in temp.

How long will a 5 m section of steel expand ($\alpha = 12\times10^{-6}$, from table 13-1, p 343) to if it is heated from 15^o to 25^{o}C?

$\Delta\ell = \ell_0\alpha\Delta T = (5)(12\times10^{-6})(25-15) = 0.0006m$ so it will be $5m + 0.0006m = 5.0006m$

Kinetic Molecular Theory: Gas molecules collide with each other and their surroundings.

1. Large number of gas molecules (N) moving in random directions and a variety of speeds.
2. They are far apart, with the separation distances being vast compared to the diameter of each particle.
3. Molecules obey laws of mechanics. They attract each other, but we ignore this since the speeds and KE are huge.
4. Collisions with each other and container walls are assumed to be perfectly elastic. Conservation of p and KE.

The higher the temperature the faster the molecules move. The particles vary in speed, so we can only measure an average. Half will be going faster and half slower than the average.

Average KE reflects this speed. $\boxed{\overline{KE} = \frac{3}{2}k_BT}$ where the Boltzman Constant is k_B = 1.38x10^{-23}. Average KE is directly proportional to temperature. But velocity is not. Since $\overline{KE} = \frac{1}{2}mv^2$ and $\overline{KE} = \frac{3}{2}k_BT$,then $\frac{1}{2}mv^2 = \frac{3}{2}k_BT$ holds true. Simply and rearrange for velocity $v = \sqrt{\frac{3k_BT}{m}}$. So we can calculate the speed of the molecules on average $\boxed{v_{rms} = \sqrt{\frac{3k_BT}{\mu}} = \sqrt{\frac{3RT}{M}}}$.

This is known as the ***Root Mean Square Velocity***. μ is the mass of a single molecule and is used with the Boltzman Constant to determine the average velocity of a single molecule. M is molar mass and is used with the gas constant to find the average speed of a large group of molecules.

Pressure: $\boxed{p = F/area}$ Measured in N/m^2 called a **Pascal**. Collisions against an object are felt as pressure.

Universal Gas Law: $\boxed{pV = nRT}$ (Static problems) or $\boxed{\frac{p_0V_0}{n_0T_0} = \frac{pV}{nT}}$ (Changing problems)

Pressure: $\boxed{1atm = 1\times10^5 Pa}$. Volume is measured in meters cubed $1000L = 1m^3$. Gas constant $\boxed{8.31\frac{J}{mol\cdot K}}$

Static Problems: One container under a specific set of conditions. What is the temperature of 30 mol of gas at 4 atm of pressure occupying 200 L? $PV = nRT$ substituting values $(4\times10^5 Pa)(0.200m^3) = (30mol)(8.315\,J/mol\cdot K)T$

Changing Problems: One container under changing conditions, or gas moved from one container to another.

Boyle's Law: Pressure and Volume are inversely proportional. As one goes up the other goes down. 200 L of gas at 4 atm of pressure is moved to a 400 *L* container, what is its new pressure? (*n* and *T* aren't mentioned, so they must stay the same, and must cancel). $\frac{p_0V_0}{n_0T_0} = \frac{pV}{nT}$ simplifies to $\boxed{P_0V_0 = PV}$. Substitute values $(4\times10^5 Pa)(0.200m^3) = P(0.400m^3)$

Charles' Law: Temperature and Volume are directly proportional. As one goes up the other goes up. 200 L of gas at 273 K are heated to 373 K, what is the new volume? (P and n aren't mentioned, so they must stay the same, and must cancel).
$\frac{p_0V_0}{n_0T_0} = \frac{pV}{nT}$ simplifies to $\boxed{\frac{V_0}{T_0} = \frac{V}{T}}$. Substituting values $\frac{0.200m^3}{273K} = \frac{V}{373K}$

Combined Law: 200 L of gas at 3 atm and 273 K are moved to a 400 L container and heated to 373 K. What is the pressure? (n isn't mentioned, so they must stay the same, and must cancel)

$\frac{p_0 V_0}{n_0 T_0} = \frac{pV}{nT}$ simplifies to $\boxed{\frac{p_0 V_0}{T_0} = \frac{pV}{T}}$. Substituting values $\frac{(3\times10^5 Pa)(0.200 m^3)}{(273K)} = \frac{P(0.400 m^3)}{(373K)}$

Heating and cooling curve

Specific Heat Capacity: c, Ability to absorb or retain heat. $\boxed{Q = mc\Delta T}$ where Q is heat.

Latent Heat: L, energy required to break intermolecular forces causing a phase / state change. $\boxed{Q = mL}$

Latent Heat of Fusion: L_F, energy to melt a solid. ***Latent Heat of Vaporization:*** L_V, energy to turn a liquid into a gas.

When phase / state is changing temperature cannot rise since the energy is required for the change.

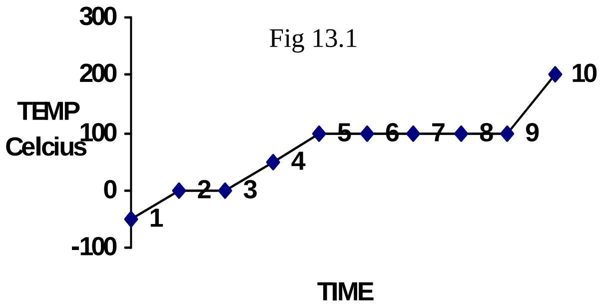


Fig 13.1

1-2	Solid	$Q = c_{solid} m\Delta T$
2-3	Melting/Freezing	$Q = mL_{fusion}$
3-5	Liquid	$Q = c_{liquid} m\Delta T$
5-9	Vaporizing/Condensing	$Q = mL_{vaporization}$
9-10	Gas	$Q = c_{gas} m\Delta T$

If more than one equation is used add the energies obtained to get the total energy.

How much energy does it take to melt 200 g of ice at $0^0 C$ and heat it to $60^o C$? It has to start as a solid, so it starts at point 2 on the graph. Melting is fusion so L is the Heat of Fusion for water from table 14-2, p. 377. Then it rises in temperature, so it must end around point 4. It is a liquid so you need the specific heat, ***c***, of liquid water in table 14-1, p. 373. (Every substance has different melting plateau, different vaporizing plateau, and different specific heats. Also each substance has different specific heats for the three principle states of matter. This values must be given in chart form or be stated in the problem.)

Melting:	$Q = mL_{fusion}$	$Q = (0.200kg)(3.33\times10^5 J/kg) = 66600J$
Heat:	$Q = c_{liquid} m\Delta T$	$Q = (4180 J/kg\cdot^o C)(0.200kg)(60^o C - 0^o C) = 50160J$
Total is	$66600 + 50160 = 116760J$	

Thermodynamics

System and environment: Think of the amount of energy in universe as constant. But, energy can be added to or subtracted from a system (engine, etc.). Energy comes from or goes to the greater universe (environment). Energy can transfer between systems. A ball colliding with another can transfer energy to that ball. Energy can change forms. If you calculate potential energy that energy can change into kinetic energy, electrical energy, thermal energy, etc.

Internal Energy: All matter has some amount of energy. Objects feel warm to the touch (internal energy: due to heat content or thermal energy of the object) caused by vibration of atoms that make up the object. If energy is added to an object it can start moving or become hotter. If energy is subtracted it can slow down or become colder. Or it can be a combination of motion and heat. Most of the beginning problems we do are done pretending that no energy is lost to the environment in the form of friction and air resistance. We often assume the energy stays in one form or that 100% is transferred when energy changes form. In real life energy is lost to the environment. Also objects become a little hotter in collisions increasing internal energy. 100% of the energy put into problems will not be available to cause motion, electricity, or power a heat engine. So many of the problems are not realistic. They ask for a theoretical unattainable maximum assuming no energy loss or perfect energy transfer.

Heat Transfer: Heat transfer occurs in one of three ways. **Conduction**: Objects touch transferring energy.
Convection: Fluids (gases & liquids) carry heat as they flow. Hot air or water rises. Convection currents.
Radiation: Electromagnetic radiation, as in infrared frequencies of light. Why does a thermos have an inner container separated from the outer by a vacuum? Why is the inside reflective?

First Law of Thermodynamics: Statement of conservation of energy. A ball falls turning PE into KE. It hits the ground causing the molecules to vibrate. The KE is transferred to internal energy and the ball comes to a stop. Have you ever seen the reverse? If internal energy caused the ball to rise, energy would still be conserved, so why isn't it reversible.

$\boxed{\Delta U = Q + W}$. ΔU : change in the system's internal energy. Q : heat added. W : work done on system (i.e. added).

+Q: heat added to system. –Q: heat lost by system. +W: work done on system. –W: means work performed by system.

There are four basic processes that affect the outcome of the first law. These can also be visualized as pV diagrams.

Adiabatic: No heat enters or leaves system, then system is said to be perfectly isolated from its environment. The process happens so rapidly so that heat does not have time to flow in or out of the system. If a gas filled cylinder is compressed very rapidly, heat won't have time to escape. If $\boxed{Q=0}$ is substituted into the first law $\boxed{\Delta U = 0 + W}$ then $\boxed{W = \Delta U}$.

Isothermal $\boxed{\Delta T = 0}$ **:** Temperature of a system does not change during a process. ***The process proceeds so slowly that temperature rise is negligible.*** Compress a gas filled cylinder so slowly that molecules don't speed up and temperature will not rise.

Remember, internal energy is directly affected by temperature $\boxed{\overline{KE} = 3/2\, k_B T}$.

If $\boxed{\Delta T = 0}$, then $\boxed{\Delta U = 0}$.

If this is substituted into the first law $\boxed{0 = Q + W}$, then $\boxed{W = -Q}$.

When T is constant Combined Gas Law reduces to Boyle's Law

$$\boxed{\frac{p_0 V_0}{T_0} = \frac{pV}{T}} \quad \boxed{p_0 V_0 = pV}$$

It graphs as a hyperbolic function. Every T has its own isothermal curve. Higher temperature curves are farther from origin.

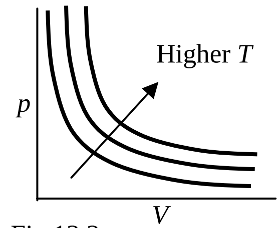


Fig 13.2

Isobaric $\boxed{\Delta p = 0}$ **:** Pressure of a system does not change during a process.

Pressure $\boxed{p = F/Area}$

Multiply by a change in volume

$$\boxed{\frac{F}{Area\left(m^2\right)} \cdot \Delta V\left(m^3\right) = F \cdot d\left(m\right) = W}$$

So $\boxed{W = -p\Delta V}$

$\boxed{W = -p\Delta V}$

$\boxed{W = -p\left(V - V_0\right)}$ which implies that ***Work is the area under the pV diagram, just as Work is the area under the Fd curve.***

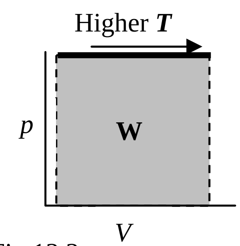


Fig 13.3

Isovolumetric (Isochoric) $\boxed{\Delta V = 0}$ **:** Volume of a system does not change during a process.

The piston does not move so there is no force through a distance. No work.

If $\boxed{\Delta V = 0}$ is substituted into $\boxed{W = -p\Delta V}$ then $\boxed{W = 0}$

There is no area under the curve shown to the right.

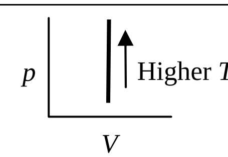


Fig 13.4

pV Diagrams and Heat Engine Cycles

T **at any point:** Use Ideal Gas Law to find temperature at any point $\boxed{pV = nRT}$

Process AB is Isobaric: $\boxed{W = -p\Delta V}$ and $\boxed{Q = nc_p \Delta T}$

Process BC is Isovolumetric: $\boxed{W = 0}$ and $\boxed{Q = nc_V \Delta T}$ and $\boxed{\Delta U = nc_V \Delta T}$

Process CA isothermal: $\boxed{W = -Q}$

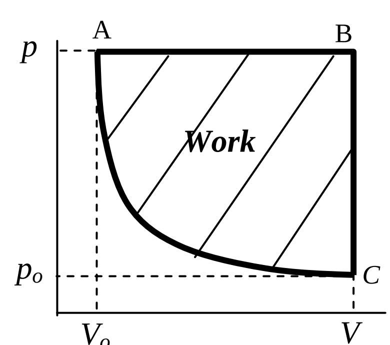


Fig. 13.5

Entire Process ABCA: In the entire process you return to point **A**. The significance of this is that you are back to the starting temperature. This means $\boxed{\Delta T = 0}$ for the process, so $\boxed{W = -Q}$ for the entire process. ***Graphically the work for an entire process is the area bounded by all the curves.***

Carnot Engine *pV* Diagram

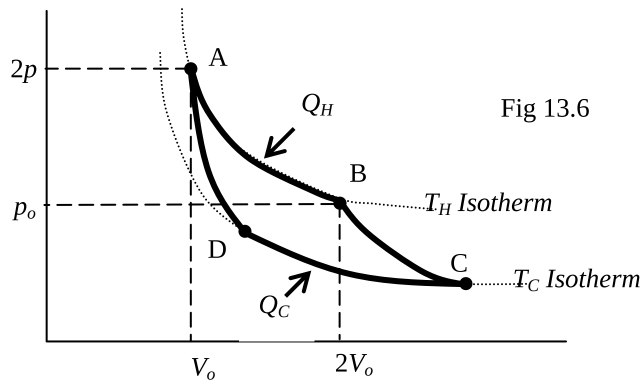


Fig 13.6

Hypothetical engine that runs equally well forward and backward. It operated between temperatures (isothermal lines). During the adiabatic phases it exchanged heat at hot temperature Q_{hot} and at cold temperature Q_{cold}.

T at any point: $\boxed{pV = nRT}$

Process AB isothermal: $\boxed{W = -Q_H}$

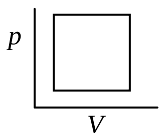
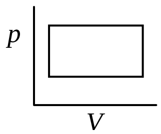
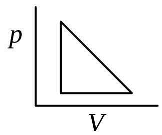
Process BC adiabatic: $\boxed{W = \Delta U}$

Process CD isothermal: $\boxed{W = -Q_C}$

Process DA adiabatic: $\boxed{W = \Delta U}$

Entire Process ABCDA: Return to point A and the starting temperature. This means $\boxed{\Delta T = 0}$ for the process, so $\boxed{W = -Q}$ for the entire process. ***Work of entire process is the area bounded by all the curves.***

Other *pV* Curves

			Any points that fit the equation $\boxed{p_0V_0 = pV}$ are on the same isothermal line. All horizontal lines are isobaric $\boxed{W = -p\Delta V}$. All vertical lines are isovolumetric $\boxed{W = 0}$. The work of a single process is the area under the curve. The work of an entire process in the area bounded by the curve.

Heat Engines: Mechanical energy can be obtained from thermal energy, only when heat is allowed to flow from a high temperature to a low temperature. Example: steam engines, internal combustion engines, and human respiration. A piece of cold wood has high thermal energy (fuel) that can be burned. This is true for gasoline and the food you eat. If this energy can be tapped it can do work. There is always a by-product: the hot exhaust of a car and human body heat are the by-products or exhaust.

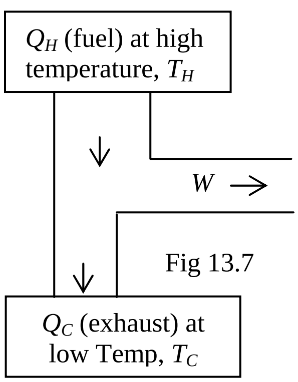


Fig 13.7

Efficiency: ratio of useful work done to the heat input.

$$\boxed{e = \left|\frac{W}{Q_H}\right|} = \frac{|Q_H| - |Q_C|}{|Q_H|} \qquad = 1 - \left|\frac{Q_C}{Q_H}\right|$$

Carnot Efficiency: an unattainable maximum theoretical efficiency. Carnot imagined a gas cylinder moved from hot to cold and expanding and contracting. He also imagined that all the processes could be done **reversibly**. Real processes involve turbulence thus making reversal along the same path impossible. Even under Carnot's ideal circumstances 100% efficiency is impossible.

$$\boxed{e_c = \frac{T_H - T_C}{T_H}} \qquad = 1 - \frac{T_C}{T_H}$$

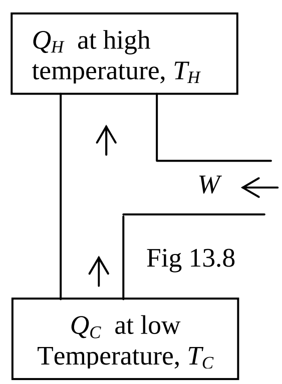


Fig 13.8

Second Law Revisited: No device is possible whose sole effect is to transform an amount of heat completely into work.

Refrigerator: (Heat Pump diagrammed to the right) Pumps heat from one area to another. Work needs to be added. But, what's happening to Q_H? It's getting bigger. So overall a refrigerators actually produce heat. As an example, in the process of lowering the inside air volume 40 degrees they may raise an equal volume of outside air by 100 degrees.

Entropy: A measure of how much energy or heat is unavailable for conversion to work. Heat is sometimes called the graveyard of energy. As Carnot showed, not all the heat energy can be used to do work. Entropy is thought of as disorganization. As we use more energy we get more heat as a by-product. Look at oil. It is composed of large well organized molecules. When we burn it to produce work, we get smaller less organized molecules and heat. The small gas by-products of combustion, CO_2 and H_2O, are more difficult to burn and extract further work from. Gases are less organized than liquids, which are less organized than solids. Perhaps all highly organized molecules will turn into small atoms of gas evenly distributed throughout the universe and they may eventually reach an even average temperature. If there is no heat difference there can be no work. So life and machines can't exist. This is called the heat death of the universe.

Second Law Re-Revisited: Natural processes to move toward a state of greater disorder (entropy). The entropy of an isolated system never decreases. Entropy can only really stay the same for idealized (reversible) processes. So it always increases. The total entropy of any system and the environment increases as a result of any natural process.

Unit 3 Electricity and Magnetism

3–14 Electric Charge, Force, and Fields

Electrostatics: Involves the dynamics and interactions between charges (q or Q) and electric fields (E). This causes force (F), electric potential (V), and electric potential energy (U_E) between charged particles and charged plates. It is considered static since the charges are not transferred between objects.

Point Charge (q): A quantity of charge that can be reduced to a single point. A charged sphere, where all of the charge acts as though it is located in the center, is similar to gravity where all of the gravity is mathematically located at a planets center. Individual protons and electrons and clusters of these particles are point charges.

Charged Plates (Q): Two flat conductive surfaces separated by a distance can act as a simple charge storage devise. The large amount of charge and energy they hold is distributed over the plates and cannot be localized to a single point. One way to represent a lot of small q's is with a large Q. While q and Q are interchangeable in equations, r and d are not. ***Equations with r in them pertain to spherical point charges, and equations with d in them are used for charged plates.***

Charge is Conserved: Charges cannot be created or destroyed.

Charge is Quantized: Comes in set quantities. All charges are a multiple of the charge on an electron.

Electricity vs. Gravity: They are both fields and share many characteristics, but have a few critical differences. The charts below compare these two very important field forces to one another.

Gravitational Fields	Electric Fields
• F_g, Force of gravity is caused by gravitational fields. • m, masses generate gravitational fields. • g is the gravity field strength.	• F_E, Force of electricity is caused by electric fields. • q, charges generate electric fields. • E is the electric field strength.
Newton's Law of Universal Gravitation **Force of gravity between two masses.** Force between masses that attract is equal and opposite, both masses pull on each other with this same value. $\boxed{F_g = G\frac{m_1 m_2}{r^2}}$ $G = 6.67\times10^{-11}\,\frac{N\cdot m^2}{kg^2}$	**Coulomb's Law** **Force of electricity between two charges:** Force between charges that attract/repel is equal and opposite, both charges pull/push on each other with this same value. **Electric charges attract & repel** $F_E = k\frac{q_1 q_2}{r^2}$ $\boxed{F = \frac{1}{4\pi \epsilon_0}\frac{q_1 q_2}{r^2}}$ $k = \frac{1}{4\pi \epsilon_0} = 9\times10^9\,\frac{N\cdot m^2}{C^2}$ $\epsilon_0 = 8.85\times10^{-12}\,C^2/N\cdot m^2$
Force of gravity on a mass in a gravitational field. $F_g = mg$	**Force of electricity on a charge in an electric field.** $F_E = qE$ $\boxed{E = \frac{F}{q}}$
Combine above equations and simplify $mg = G\frac{m_1 m_2}{r^2}$ $\boxed{g = G\frac{m}{r^2}\ (not\ given)}$ **This is an important often used formula!**	**Combine above equations and simplify** $qE = k\frac{q_1 q_2}{r^2}$ $\boxed{E = k\frac{q}{r^2}\ (not\ given)}$ **This is an important often used formula!**
What if you • Double one mass?: F_g and g double. • Double r ?: F_g and g are 1/4. • Halve r ?: F_g and g are quadrupled Changes to Mass are directly proportional. Changes in distance involve the ***Inverse Square Law***.	What if you • Double one mass?: F_E and E double. • Double r ?: F_E and E are 1/4. • Halve r ?: F_E and E are quadrupled Changes to Charge are directly proportional. Changes in distance involve the ***Inverse Square Law***.

Example 14-1: Superposition (Adding electric field vectors)

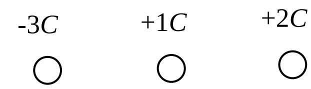


Fig 14.1a

What is the force of electricity on a +1 *C* charge located half way between a –3 *C* charge and a +2 *C* charge separated by 2 *m*, as shown in Fig 14.1a?

Find F_E for the –3 & +1 chagres, and then the +2 & +1 charges. Then add the two F_E vectors to find ΣF_E.

$F_E = k\dfrac{q_1 q_2}{r^2}$ $\qquad F_E = 9\times10^9 \dfrac{N\cdot m^2}{C^2}\dfrac{(3C)(1C)}{\left[1/2(2m)\right]^2} = 27\times10^9 N$ –3 charge attracts the +1 charge to the left, ***negative direction***.

$F_E = 9\times10^9 \dfrac{N\cdot m^2}{C^2}\dfrac{(2C)(1C)}{\left[1/2(2m)\right]^2} = 18\times10^9 N$ +2 charge repels the +1 charge to the left, ***negative direction***.

Note that the minus sign on the –3C charge was not included in the equation. The equation solves for the magnitude of the vector, which has an absolute value. Do not include minus signs on charge when solving for force electric. The minus signs are used to determine whether the object will repel or attract. To the right of each equation is an explanation of how the signs on charges are used. Once the sign on each vector is made then you can sum them.

Sum the vectors $-27\times10^9 N + -18\times10^9 N = -45\times10^9 N$ The negative sign means to the left $\boxed{45\times10^9 N,\ left}$.

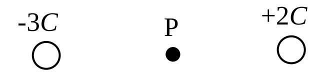


Fig 14.1b

What is the electric field strength at point P located half way between a –3 *C* charge and a +2 *C* charge separated by 2 *m*, as shown in Fig 14.1B?

Find E at point P for the – 3 and then the +2 charge. Then add the two E vectors to find ΣE.

$E = k\dfrac{q}{r^2}$ $\qquad E_{from-3} = 9\times10^9 \dfrac{N\cdot m^2}{C^2}\dfrac{(3C)}{\left[1/2(2m)\right]^2} = 27\times10^9\ N/C$

The –3 charge attracts a positive test charge to the left, ***negative direction***.

$E_{from+2} = 9\times10^9 \dfrac{N\cdot m^2}{C^2}\dfrac{(2C)}{\left[1/2(2m)\right]^2} = 18\times10^9\ N/C$

The +2 charge repels a positive test charge to the left, ***negative direction***.

Again the minus signs are not included in the magnitude calculation. They are used to determine repulsion and attraction.

Sum the vectors $-27\times10^9 + -18\times10^9 = -45\times10^9\ N/C$ The negative sign means to the left $\boxed{45\times10^9\ N/C,\ left}$.

Alternate way to find the force on the +1C charge placed at point P. We solved for the electric field strength at point P independent of a charge existing there. This is actually preferred, as once we have this value for empty space, we can easily put any charge there and find the force. If we insert a +1C charge at point P in Fig 14.1b, then we have the same scenario as Fig 14.1a. But, now that we have the electric field at point P, lets see how easy it is to find force.

$F_E = qE$ $\qquad F_E = (1C)\left(45\times10^9\ N/C\right) = \boxed{45\times10^9 C}$ This is the same answer as in the first part above.

The advantage to this second method is that you can do superposition for the electric field once, and then use it for any charge placed at that point. If you use the force superposition method, you must do a new force superposition problem for every charge placed at point P.

Example 14-2: Electric Force

A negatively charged wall repels a charged mass, attached to the wall by a string, shown in Fig 14.2a. Fig 14.2b shows the FBD for this scenario, and a diagram of the three vectors added tip to tail. Note that the three vectors sum to zero. The object is not moving and therefore the sum of force should be zero.

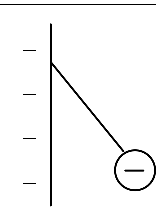


Fig 14.2a

What is the force tension in the string?

$$T^2 = F_g^{\ 2} + F_E^{\ 2}$$

$$T = \sqrt{F_g^{\ 2} + F_E^{\ 2}}$$

$$T = \sqrt{mg^2 + qE^2}$$

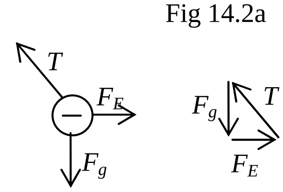


Fig 14.2b

Electric Field Lines: An imaginary way to view the electric field, like the way imaginary contour lines are used on maps. In mapping, steep slopes are diagrammed by drawing the contour lines closer together. In a similar fashion a concentration of any field lines (gravitational, electric, and magnetic) indicates higher field strength. Gravitational field lines are based on the direction of movement of a test mass. ***Electric field lines are based on the direction of motion of a imaginary positive test charge in an electric field.*** As a result electric field lines leave positive charges and enter negative charges. Every charged object generates an electric field. Field lines ***leave and enter surfaces perpendicular to the surface***. The field lines around a proton, an electron, between a proton and an electron, and between charged plates as shown in Fig 1.2. The field direction at any point in the diagrams below is easy to identify. Simply imagine a positive charge at the point in question and ask what direction it will go. ($+x$, $-x$, $+y$, $-y$, etc.). When dealing with a curved portion the field is tangent to a curved surface.The field is solved for a point ***P*** in each case.

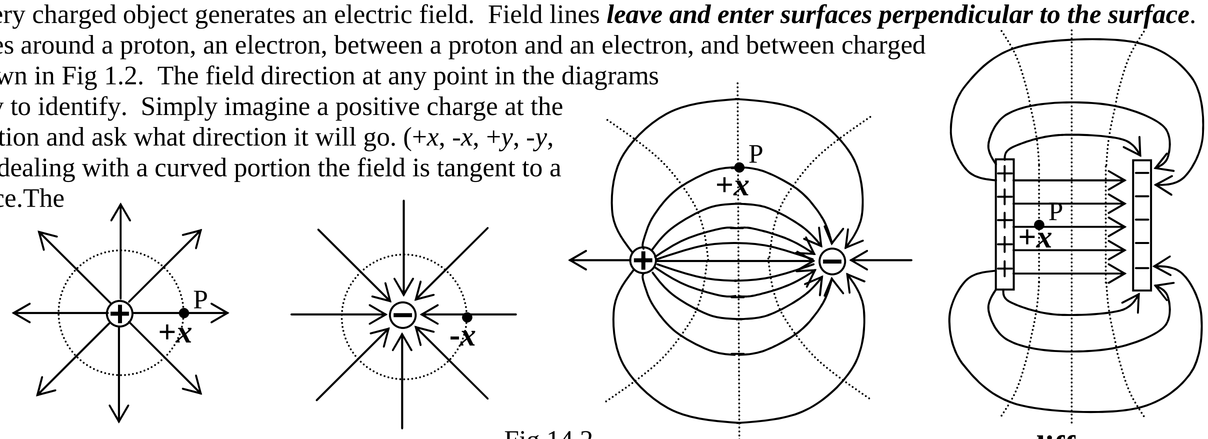


Fig 14.2

Potential difference or Voltage: $\boxed{V = Ed}$ In gravity it would be gh. We never worked with this term since g is weak, and does not change significantly over short distances. We normally concern ourselves only with changes in h, unless we leave a planet for some point in space, or we visit a large gravitational source such as a black hole. Electricity on the other hand is billions and billions of times stronger than gravity. A change in distance d (since no clear h exists in electricity) is accompanied by a significant change in electric field E as well. The electrical equivalent of gh is potential Ed and is commonly called voltage where $V = Ed$.

Equipotential Lines: Lines of equal electric potential (a component of potential energy). If a charge moves along an equipotential line it does not change its potential or voltage. This is like hiking in the mountains along the contour lines where your height and gravity above sea level don't change. Equipotential lines are always perpendicular to field lines. The dotted lines in the diagrams above are some examples of the many possible equipotential lines. The electric field E, is a component of potential ($E = V/d$) and will not change if potential V and distance d (or r) do not change. If you find the electric field strength at P in the two left diagrams it will be the same at any equal distance from the center of the point charge. At this distance an equipotential sphere, with radius r exists where Ed stays the same. This is why r is used for distance around point charges. The potential V and electric field E are constant everywhere in a sphere of radius r. In the right most diagram, we assume that the electric field between the plates is uniform. So if potential V and distance d are not changing along the straight dotted line between and parallel to the plates, then the electric field $E = V/d$ on this line is equal to that at point P. So potential and electric field depend on the distance d between the plates. Use formulas with d for charged plates. The curving electric field in the combined proton / electron diagram and outside the plates in the diagram to the right is not equal in strength. Remember equipotential lines mean that potential is the same, not necessarily that the electric field strength is equal. The electric field is only equal along equipotential lines where distance (d or r) from the charge/charges is unchanging.

Faraday Cage: Any enclosed metal structure, even one made of chicken wire, acts as a Faraday Cage. Charges piles up on the outer surface of a metal enclosure. Due to a combination of q and r at any point within a box, cylinder, sphere, etc. the electric field is zero inside. This is why you are not electrocuted in a car or airplane if it is struck by lightening. Shielding in electronic means Faraday Cage.

Conductors: Substances in which charges can move freely. When a conductor is charged the individual charges pile up over the outside surface area of a conductor. ***The electric field inside a charged conductor is zero.*** Outside the surface the electric field drops according to the inverse square law. The relationship of electric field to radius, in a conductor, is shown in Fig 14.3.

Insulator: Substance that does not allow charges to move freely. Insulators can become charged. A plastic comb run through your hair is one example. But, the charges do not distribute over the surface area. Only the area affected becomes charged.

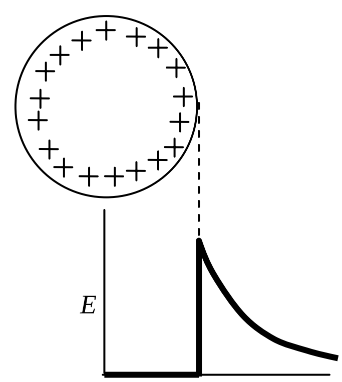


Fig 14.3

3–15 Gauss's Law

Note: Gauss's Law is needed for AP Physics C: physical science majors, calculus based.

Flux: The amount of a field (gravity, electricity, or magnetism) passing through a defined area of space. $\phi = (Field\ \ Strength)(Area)$. Fields are visualized as lines (vectors) in space. The closer the lines are to each other the stronger the field in that region of space. Flux can then be thought of as the number of field lines passing through an area. The more lines passing through the area the greater the flux is. Flux is also greatest when the field lines pass through the area perpendicular to it. In Fig 15.1 no field lines pass through the area on the left. The field lines skim above and below it. In the area at an angle, in the middle, three of five field lines pass through, while in the identical area to the right five field lines pass through. The largest available area is exposed to the field when the area is perpendicular to the field. In this section we will deal with the electric flux. But, the same logic can be applied to gravitational flux and magnetic flux.

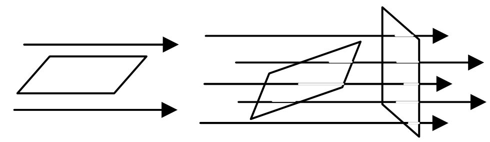


Fig 15.1

Electric Flux: Amount of electric field in an area of space. $\phi_E = \mathbf{E} \cdot \mathbf{A}$. This is a dot product of vectors, so $\phi_E = \mathbf{EA}\cos\theta$. But, this appears not to make any sense. Looking at the right area in Fig 15.1 we see that the area and electric field are 90^o apart. Using the above formula would result in zero flux, in a configuration that was just described as having maximum flux. But, in physics ***whenever you measure angles in relation to an area a normal is used***. A normal is a line drawn perpendicular to the surface. This convention is used in force normal, it is used in flux, and it is used in optics for lenses and mirrors. Fig 15.2 shows the relationship between the area, the normal, and the electric field. Normal lines are drawn as dashed lines perpendicular to the surface. The angle between the normal and the field is 0^o and this results in maximum flux.

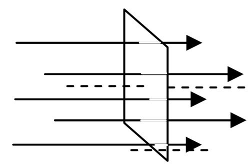


Fig 15.2

Flux Through a Cube (Closed Surface): The cube in Fig 15.3 has an electric field passing through it. There is zero flux through the top, bottom, front, and back of the cube, as the field does not pass through them. Through the left surface, the electric field is to the right and the normal is to the left. The angle between them is 180^o. This results in a negative flux. ***When ever a field enters an object the flux is negative***. On the right side of the cube, the field is in the same direction as the normal. The angle between them is 0^o. This results in a positive flux. ***When ever a field leaves an object the flux is positive***. The left area equals the right area, and the number of field lines passing through both areas is the same. The flux in (negative) cancels the flux out (positive). The **net flux** is zero.

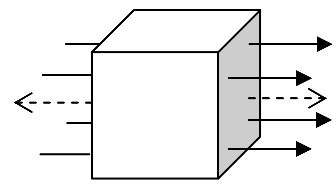


Fig 15.3

Irregular Closed Surfaces: If the area is irregular, then integral calculus must be used to find flux $\phi_E = \oint \mathbf{E} \cdot d\mathbf{A}$. The circle is added to the integral symbol to denote integration over a closed surface, such a sphere or the irregular shape in Fig 15.4. You are basically finding the flux over various small sections of area and summing them together.

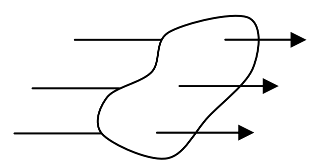


Fig 15.4

Gaussian Surface: Gauss was concerned with the net flux passing through a closed (**gaussian**) surface. Gaussian surfaces are any closed surfaces in space that are useful to analyze the problem. If you are looking at a point charge (spherical) then surround it with a spherical gaussian surface. When looking at a section of wire surround it with a cylindrical gaussian surface. Just like field lines it does not exist, it is a way to analyze invisible fields.

Analyzing a Proton Using a Gaussian Surface: In Fig 15.5 we analyze the electric field, at a distance r, surrounding a proton. ***The diagram looks circular, but this is really a spherical arrangement.*** Picture a gaussian sphere at a radius r surrounding the proton, use $\boxed{\phi_E = \oint \mathbf{E} \cdot d\mathbf{A}}$.

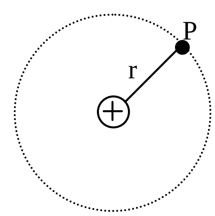


Fig 15.5

The symmetrical nature of this gaussian surface allows us to move E outside of the integral, $\phi_E = \mathbf{E}\oint d\mathbf{A}$. We know the surface area of a sphere, and can avoid its integration $\phi_E = \mathbf{E}(4\pi r^2)$.

Previously we solved for the electric field at a point point $\mathbf{E}=k\dfrac{q}{r^2}$, and we know that $k = \dfrac{1}{4\pi \in_0}$.

Combining the three previous equation leads to $\phi_E = \left(\dfrac{1}{4\pi \in_0}\dfrac{q}{r^2}\right)(4\pi r^2)$, which simplifies to $\boxed{\phi_E = \dfrac{q}{\in_0}}$. Note that the flux is the same at any radius r. The flux equation gets larger by the square of r, but the electric field decreases by the square of r. When the equations combine r^2 cancels. The area grows but the proton's electric field is spread over a larger area. Thus the effect of increasing/decreasing r cancels. So $\phi_E = \oint \mathbf{E} \cdot d\mathbf{A}$ and $\phi_E = \dfrac{q}{\in_0}$. Therefore, $\boxed{\phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = \dfrac{q}{\in_0}}$

Gauss's Law: Gauss introduced the gaussian surface concept, and derived the previous formula $\boxed{\phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = \frac{q}{\epsilon_0}}$. He also showed that this relationship works for any closed surface. It is easy to work with symmetrical objects that have known surface areas since you can avoid the actual integration. But, if the integration is performed over any irregular closed surface the relationship can be shown to hold true.

There are two ways to use a gaussian surface.

1. The surface can enclose at net charge. The proton in Fig. 15.5 was such an example. In these cases the above formula holds true. Stated formally: ***The net flux through any closed surface surrounding a point charge q is given by*** $\boxed{\phi_E = \frac{q}{\epsilon_0}}$**.** This can be a single proton or a single electron. It can also be a group of charges. You must work with the net charge enclosed by the surface. The logic is the same as that shown for the lone proton on the previous page.
2. The surface can enclose zero net charge. The charges could be outside the surface. In Fig. 15.6 the proton is outside the gaussian surface. Note that every field line entering the gaussian surface (negative flux) eventually exist the gaussian surface (positive flux) at another point. The positives and negative cancel. The net flux is zero. ***The net electric flux through a closed gaussian surface that surrounds no charge (or zero net charge) is zero***. Looking back at the cube in Fig. 15.3 we see that the net flux is zero through the cube (a closed gaussian surface). This holds true even if the surface is highly irregular.

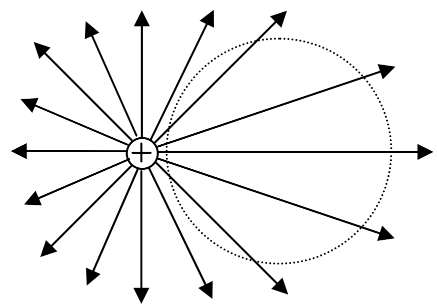


Fig 15.6

Example 15-1: Electric Flux Through Various Surfaces.

Fig 15.7 shows various gaussian surfaces drawn around several charges.

Find the electric flux through each surface.

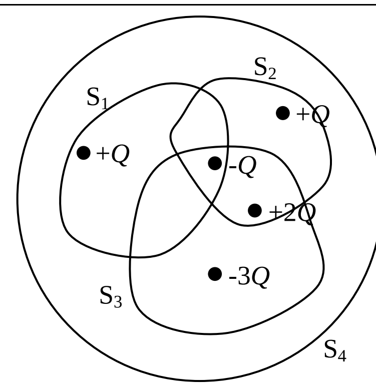


Fig 15.7

S₁: holds a $+Q$ and $-Q$ charge. The net charge is zero.

$$\phi_E = \frac{q}{\epsilon_0} \qquad \phi_E = \frac{(+Q)+(-Q)}{\epsilon_0} \qquad \boxed{\phi_E = 0}$$

S_2: holds a $+Q$, $-Q$, and $+2Q$ charge. The net charge is $+2Q$.

$$\phi_E = \frac{q}{\epsilon_0} \qquad \phi_E = \frac{(+Q)+(-Q)+(+2Q)}{\epsilon_0} \qquad \boxed{\phi_E = \frac{+2Q}{\epsilon_0}}$$

The positive flux indicates that the electric field through surface S_2 is moving outward, as it should from a positive charge.

S_3: holds a $+2Q$, $-Q$, and $-3Q$ charge. The net charge is $-2Q$.

$$\phi_E = \frac{q}{\epsilon_0} \qquad \phi_E = \frac{(+2Q)+(-Q)+(-3Q)}{\epsilon_0} \qquad \boxed{\phi_E = \frac{-2Q}{\epsilon_0}}$$

The negative flux indicates that the electric field through surface S_3 is moving inward, as it should from a negative charge.

S_4: holds a $+Q$, $+Q$, $+2Q$, $-Q$, and $-3Q$ charge. The net charge is zero.

$$\phi_E = \frac{q}{\epsilon_0} \qquad \phi_E = \frac{(+Q)(+Q)(+2Q)+(-Q)+(-3Q)}{\epsilon_0} \qquad \boxed{\phi_E = 0}$$

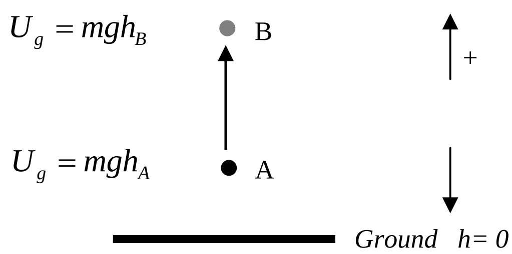
When viewed close up a collection of charges may seem complicated. Look at the three charges inside surface S_2. Close up we see $+Q$, $-Q$, and $+2Q$ charges. If we drew the electric field lines it would be a very complicated three-dimensional diagram. However, if we stepped way back, we would see a very small point with a +2Q net charge. At a great distance it would look like the electric field of a point charge. So when we draw a gaussian surface around these three charges we are analyzing them as a single point charge. The field lines that leave the system are those left over after tying the charges together.

3–16 Electric Potential

Electric Energy Compared to Gravitational Potential Energy: Electric fields created by charged plates (Q) are directed from positive to negative. Electrons ($-q$) or protons ($+q$), which have their own electric fields, experience a force when placed between plates. The positive proton follows the electric field, while the electron moves opposite the field.

Gravitational Fields

There is only one kind of mass and it only attracts. In the following scenarios a mass will be moved in a gravitational field created by the planet earth.



- Gravitational Field acts from + sky to – ground.
- ***<u>Masses naturally move in the field direction.</u>***
- To lift a mass opposite the field direction, from A to B, you must add energy. You must do $+W$.
- Work Energy Theorem states that $W = \Delta\,Energy$.
- Point A has less potential energy than point B.
- Moving from A to B increases U_g.
- The increase (change) in U_g equals the work done.

Masses only attract each other.
They only attract and thus fall toward each other.

But gravity is very weak, billions and billions of times weaker than electricity. <u>So in electric force problems gravity is ignored</u>. It is there, but it is mathematically irrelevant. One exception is the Milikan Oil Drop Experiment, where he placed excess charge on oil drops. The oil drops have significant mass compared to electrons & the gravity on the oil drops (not the electrons) matters.

m	causes gravity
g	measure of field strength
h	distance above ground

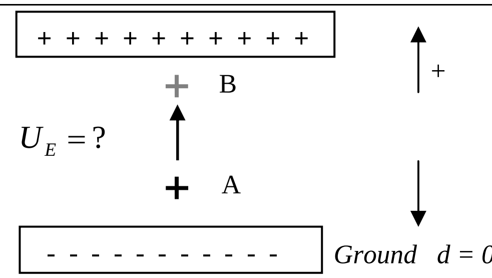
Gravitational Potential Energy $\boxed{U_g = mgh}$

In Newtonian Mechanics there is no expression similar to Electric Potential. In electricity $V = Ed$, but $? = gh$

The other equation for potential energy of gravity was solved in the mechanics section on gravity, as follows:
$W_g = \Delta U_g = F_g \Delta r$. Set the initial displacement as zero and it simplifies to $U_g = F_g r$. Use this with $\boxed{F_g = -G\dfrac{m_1 m_2}{r^2}}$ to get $\dfrac{U_g}{r} = -G\dfrac{m_1 m_2}{r^2}$. This simplifies to $\boxed{U_g = -G\dfrac{m_1 m_2}{r}}$.

Electric Fields

Two types of charge that repel and attract. ***<u>Electrical theory is based on what positive charges do.</u>*** The following scenarios show a positive charge moved in an electrical field.



- Electric Field acts from + plate to – plate.
- ***<u>Positive charges naturally move in the field direction.</u>***
- To move positive charge opposite the field direction, from A to B, you must add energy. You must do $+W$.
- Work Energy Theorem states that $W = \Delta\,Energy$.
- Point A has less potential energy than point B.
- Moving from A to B increases U_E.
- The increase (change) in U_E equals the work done.

But, Electricity has positives and negatives.
Opposite charges attract and positive charge in electric fields act just like masses in a gravity fields.
Negative charges follow the same rules but move in the opposite direction. They fall up, so to speak. They are repelled by the negative ground and attract to the positive.
In addition to this like charges repel. So while it is similar to gravtiy there is added complexity. The math is the same, but the directions of fields, forces, and velocities can be confusing.

q	causes electricity
E	measure of field strength
d	distance above negative plate (ground)

Electric Potential Energy $U_E = qEd$

New quantity in electricity $\boxed{E = \dfrac{V}{d}}$ so $V = Ed$

Substitute $V = Ed$ into the electric potential energy equation and you get its final form
$\boxed{U_E = qV}$.

Combine $U_E = qEd$ & $E = k\dfrac{q}{r^2}$ to get

$U_E = q\left(k\dfrac{q}{r^2}\right)d$ r and d cancel since both measure distance. $\boxed{U_E = qV = k\dfrac{q_1 q_2}{r}}$

Gravitational Fields	Electric Fields
Work: $W = \Delta U_g = mg\Delta h$	**Work**: $W = \Delta U_E = q\Delta V$
	Electric Potential Due to Several Charges Combine $E = k\dfrac{q}{r^2}$ & $E = \dfrac{V}{d}$ to get $\dfrac{V}{d} = k\dfrac{q}{r^2}$ r and d cancel since both measure distance. Simplifies to $V = k\dfrac{q}{r}$ and is written as $\boxed{V = k\sum_i \dfrac{q_i}{r_i}}$

Electric Potential is easily confused with Electric Potential Energy. Even though they sound similar they are different variables. Unfortunately there are also a number of expressions for each variable. Know the following!
V: ***Potential, Electric Potential, Potential difference, and Voltage***.
U_E: ***Potential Energy, and Electric Potential Energy***.
You can tell if it is $\boldsymbol{U_E}$, as it has the critical word ***Energy***.

Example 16-1: Electric Potential Due to Several Charges

What is the electric potential at point P located half way between a –3 *C* charge and a +2 *C* charge separated by 2 *m*, as shown in Fig 16.1?

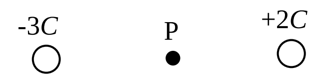


Fig 16.1

Potential is not a vector. This time just sum the quantities in a very straight forward manner, using the equation below. ***Since it is not a vector the minus signs on the charge are included in the equation.***

$$V = k\sum_i \frac{q_i}{r_i} \qquad V = 9\times10^9\,\frac{N\cdot m^2}{C^2}\left(\frac{-3C}{1m} + \frac{+2C}{1m}\right) = -9\times10^9\,\frac{N\cdot m}{C} = \boxed{-9\times10^9 V}$$

Example 16-2: Electric Field, Potential, and Potential Energy

A $+2\times10^{-6}$C charged mass is located 2 m to the left of point P, as shown in Fig. 16.2a

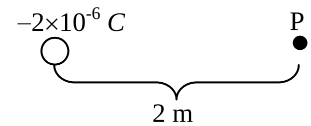


Fig 16.2a

Solve for the Electric Field at point P in Fig 16.2a.

$$E = k\frac{q}{r^2} \qquad E = \left(9\times10^9\right)\frac{\left(2\times10^{-6}\right)}{(2)^2} = \boxed{4.5\times10^3\ N/C,\ \text{left}}$$

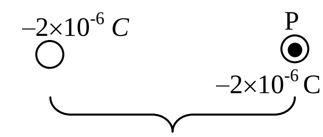


Fig 16.2b

Remember, electric field is a vector. Do not include the minus sign on the, -2C charge, in the math when solving for the magnitude of the field. Use the minus on the charge to establish the direction of the field. Electric field is directed toward negative charges (ground), just as the gravity field is directed downward (-) toward the ground.

Solve for the Electric Potential at point P in Fig 16.2a.

$$V = k\sum \frac{q_i}{r_i} \qquad V = \left(9\times10^9\right)\frac{\left(-2\times10^{-6}\right)}{(2)} = \boxed{-9\times10^3 V}$$

Voltage is not a vector. Just plug in the values, minus signs and all, as you see it. There are no worries about direction.

Another –2 C charge mass is moved to point P, as shown in Fig 16.2b. Solve for the systems Electric Potential Energy.

$$U_E = qV = \left(2\times10^{-6}\right)\left(9\times10^3\right) = \boxed{18\times10^{-2} J} \quad \text{or} \quad U_E = k\frac{q_1q_2}{r} = \left(9\times10^9\right)\frac{\left(2\times10^{-6}\right)\left(2\times10^{-6}\right)}{(2)} = \boxed{1.8\times10^{-2} J}$$

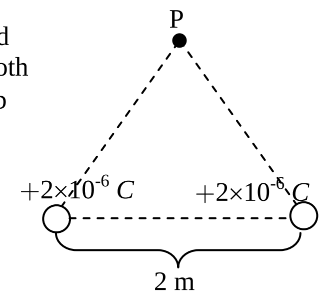
If the charged masses are released from this position, what will their final velocities be? If this potential energy is released the charges will repel and begin moving. The energy of motion is kinetic energy. Try conservation of energy, potential turning into kinetic. ***Just remember there are two charged masses, each having kinetic energy***. This problem will then require mass: both charges are identical with a mass of 3×10^{-12} kg.

$$K_{q1} + K_{q2} = U_E \qquad (2\ \text{charges})\left(\frac{1}{2}mv^2\right) = k\frac{q_1q_2}{r} \qquad v = \sqrt{\frac{kq_1q_2}{mr}} = \sqrt{\frac{\left(9\times10^9\right)\left(2\times10^{-6}\right)\left(2\times10^{-6}\right)}{\left(3\times10^{-12}\right)(2)}} = \boxed{7.75\times10^4\ m/s}$$

This final velocity is reached at infinity, where the potential energy is zero. Can you do this problem in reverse?

Example 16-3: A Two Dimensional *E*, *V*, and U_E Problem

An equilateral triangle is formed by two $+2.0\times10^{-6}$ C charges and point P, as shown in Fig 16.3a. Each side is 2 m long.



Solve for the Electric Field at point. Since this problem involves vectors that are not on the coordinate axis, vector components or a geometric trick must be used. In the component method you will solve for the E_x's and E_y's from each charge, then add them to get the resultants in both the x and y directions. Then Pythagorean Theorem them together to solve for ΣE. Fig. 16.3b shows the vector components needed and this method is shown below

The Long Way

The charges have the same magnitude and are the same distance from P. Therefore, the electric field is the same for both.

$$E = k\frac{q}{r^2} \qquad E_1 = E_2 = \left(9\times10^9\right)\frac{\left(2\times10^{-6}\right)}{\left(2\right)^2} = 4.5\times10^3$$

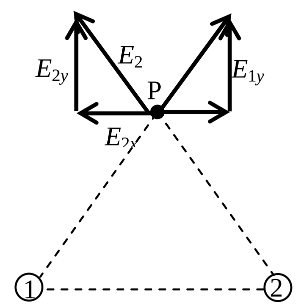


Fig 16.3b

Convert to components.

$$E_{1x} = E_1\cos\theta = 4.5\times10^3\cos60^o = +2250 \qquad E_{1y} = E_1\sin\theta = 4.5\times10^3\sin60^o = +3897$$

$$E_{2x} = E_2\cos\theta = 4.5\times10^3\cos120^o = -2250 \qquad E_{2y} = E_2\sin\theta = 4.5\times10^3\sin120^o = +3897$$

$$\sum E_x = E_{1x} + E_{2x} \qquad \sum E_y = E_{1y} + E_{2y}$$

$$\sum E_x = 2250 - 2250 = 0 \qquad \sum E_y = 3897 + 3897 = +7794$$

Then Pythagorean Theorem and Arctangent the x and y vectors to get the ΣE.

$$\sum E = \sqrt{{E_x}^2 + {E_y}^2} = \sqrt{\left(0\right)^2 + \left(7794\right)^2} = 7794\,N/C \qquad \theta = \tan^{-1}\frac{E_y}{E_x} = \tan^{-1}\frac{7794}{0}$$

this does not solve, but looking at the diagram it is apparent that the two y component vectors are pointing at 90°. $\boxed{7794\,N/C \ \ at\ 90^o}$ or $\boxed{7794\,N/C,\ +y\ direction}$

You wouldn't have known that the direction was $+y$ if you neglected to draw the diagram. In addition, drawing and looking at the diagram reveals an easy shortcut.

Short Way

Both charges are the same, and all distances are the same. So the x-component vectors are equal, as are the y-component vectors. If you imagine all the components added tip to tail it is apparent that the x-components cancel. $\sum E_x = 0$. This means that the final solution is the y-component doubled. $\sum E = E_{1y} + E_{2y} = 2E_y = 2\left(4.5\times10^3\sin60^o\right) = 7794\,N/C$. A glance at the diagram shows that this is pointing in the $+y$-direction. $\boxed{7794\,N/C,\ +y\ direction}$.

<u>Always draw a vector diagram and think about it before you do too much work.</u>

Solve for the Electric Potential at point P in Fig 16.3a.

$$V = k\sum\frac{q_i}{r_i} \qquad V = \left(9\times10^9\right)\frac{\left(+2\times10^{-6}\right)}{\left(2\right)} + \left(9\times10^9\right)\frac{\left(+2\times10^{-6}\right)}{\left(2\right)} = \boxed{1.8\times10^4 V}$$

Voltage is not a vector. Just plug in the values, minus signs and all, as you see it. There are no worries about direction.

Another +2 C charge mass is moved to point P. Solve for the Electric Potential on this new added charge.

There are three charges so the equation $U_E = k\frac{q_1 q_2}{r}$ may not help. Try $U_E = qV$. We just calculated the voltage of pressure, at point P, due to the two original charges. Now a new charge is introduced. The potential energy of this charge is a function of the voltage (from the other charges) pushing on it, and the amount of charge it has.

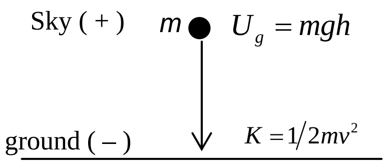
$$U_E = qV = \left(2\times10^{-6}\right)\left(1.8\times10^4\right) = \boxed{0.036J}$$

This is also the amount of work needed to push this new charge from infinity (zero potential energy) to point P.

If this charge were to be released this potential energy would be converted into kinetic energy on this one charge. $U_E = K_{1\,charge}$ **and** $qV = \frac{1}{2}mv^2$. **If all three charges are released then the potential energy converts into kinetic energy of all three, with each receiving one third.** $U_E = K_{3\,charge}$, **so** $U_E = 3K$ **and** $qV = 3\left(\frac{1}{2}mv^2\right)$.

Movement of Mass m/Charge q Parallel and Perpendicular to Gravity / Electric Fields

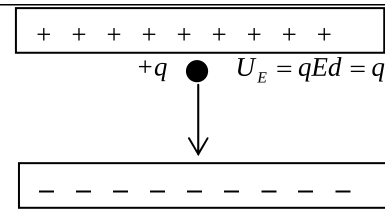
Movement Parallel to Gravitational Field



When masses fall from + to –, PE changes to KE

$$\boxed{mgh = \frac{1}{2}mv^2 \quad (not\ given)}$$

Movement Parallel to Electric Field

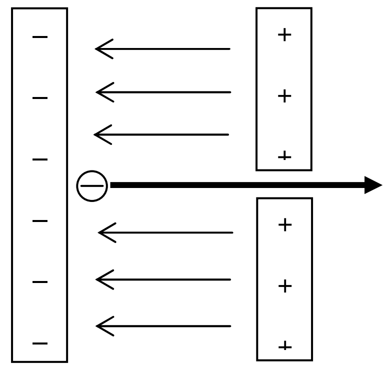


When positive charges fall from + to –, PE changes to KE

$$\boxed{qV = \frac{1}{2}mv^2 \quad (not\ given)}$$

Gravity is simple since it is directed toward any mass, and masses are of one type. They fall toward each other.

But, plates are not always up and down like gravity. And there are two types of charges. You must remember that electrical theory is based on positive charge, and that the direction of the electric field is positive to negative (the direction a positive test charge would move). If negative charges are involved then the direction of force and motion are reversed. One of the plates is often depicted with a hole in it. This is the basis of a charge gun. Charges will accelerate toward the plate with the hole, and some will pass through the opening. In our problems the field is assumed to be uniform between the plates (like gravity is assumed to be 9.8 anywhere close to Earth's surface, even though it actually changes slightly with height). And the field is assumed not to extend beyond the plates (It does but is weak). In the example to the right the plates have been rotated 90^{o} reversed and then a negative charge is used. Be prepared for anything!!!!!!!

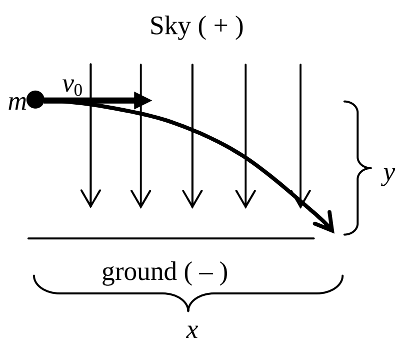


The electric field is directed **– *x***, but the negative charge does not follow the field as a positive charge would. It moves instead **+ *x***. But the equation is the same.

$$qV = \frac{1}{2}mv^2$$

Note: If the charges are electrons and protons, there will be a difference in final velocities since electrons have less mass than protons.

Projectile Motion



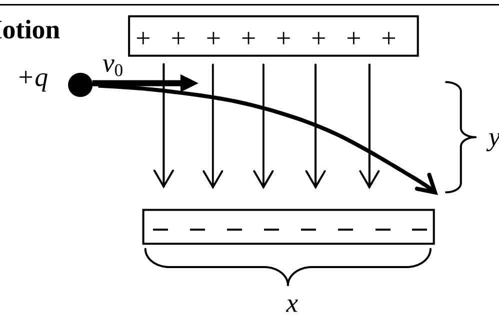
In gravity the acceleration is known, g = 9.8m/s^2

Horizontal launch (as drawn): $v_{0x} = v_0$ and $v_{0y} = 0$

$$x = x_0 + v_{0x}t + \frac{1}{2}at^2 \qquad y = y_0 + v_{0y}t + \frac{1}{2}gt^2$$

$$\boxed{x = v_0 t \quad (not\ given)} \qquad \boxed{y = \frac{1}{2}gt^2 \quad (not\ given)}$$

Projectile Motion



The acceleration of electricity must be solved as it is different for every set of plates.

$$\sum F = F_E \qquad ma = qE \qquad a = \boxed{\frac{qE}{m}}$$

Horizontal launch (as drawn): $v_{0x} = v_0$ and $v_{0y} = 0$

$$x = x_0 + v_{0x}t + \frac{1}{2}at^2 \qquad y = y_0 + v_{0y}t + \frac{1}{2}at^2$$

$$\boxed{x = v_0 t \quad (not\ given)} \qquad \boxed{y = \frac{1}{2}at^2 \quad (not\ given)}$$

No g (electricity is so strong gravity is ignored). So use we use a which is the acceleration caused by the electric field.

Example 16.4: Combining the parallel and perpendicular electric fields into one problem

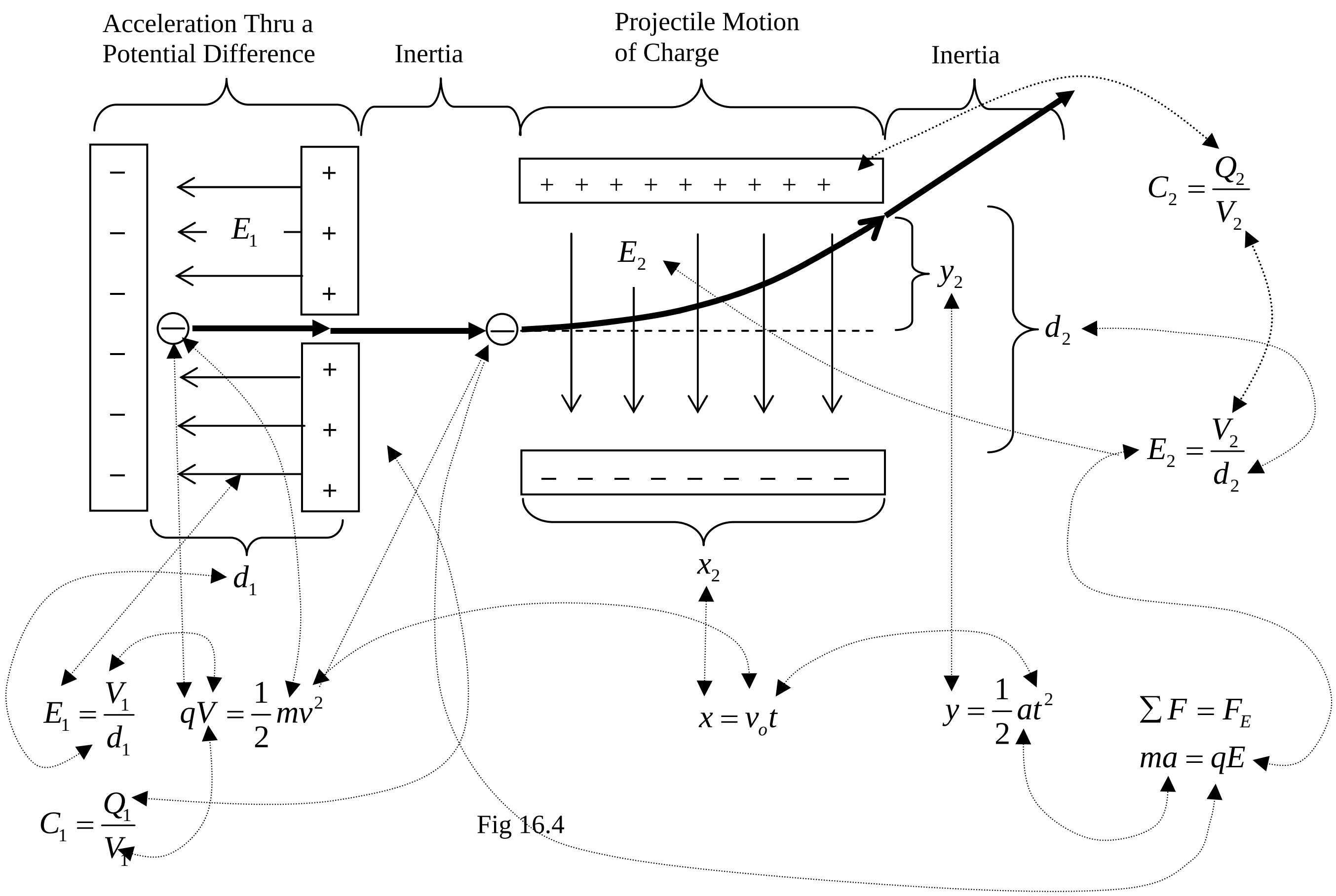


Fig 16.4

In this scenario

An electron is accelerated through a potential difference by the plates to the left. Remember when any field acts parallel to an object it will either accelerate or decelerate the mass or charge. The field is always assumed to be uniform between the plates (same strength everywhere, and it is assumed not to extend beyond the plates). Upon exiting the first set of plates the electron moves at constant velocity due to its inertia. Then it enters a second set of plates. This time the field is perpendicular. Fields that are perpendicular to motion cause either projectile motion or circular motion depending on whether the field is linear or radial. The electron above experience projectile motion due to the linear field, although it appears upside down compared to a gravity problem. The electron continues in a straight path due to its inertia once it leaves the plates.

Given: The electric field E_1 of the first plates and the distance between them d, or the charge on the plates Q_1 and the capictance of the plates C_1.
The mass m_e and charge q_e of an electron accelerated by the plates.
The length x_2 of the second set of plates, the vertical displacement y_2 of the electron, d_2 the distance between the plates.
Find: The electric field E_2 and potential difference V_2 of the second set of plates.
The speed v_f of the electron leaving the second set of plates.
Be able to draw the path of the electron from beginning to end.
Don't Confuse small v for velocity and capital V for voltage.

Try this on for size: What is E_2 in terms of E_1 and any of the other above variables and constants that are relevant?

$$\boxed{E_2 = \frac{4E_1 d_2 y_2}{x_2^{\,2}}}$$

Anything is possible: **The firing:** Plates can be reversed to fire a proton. Plate charge may not be marked, but may be a question. The electric field will not be drawn, instead they will require you to draw the field or to state its direction according to the coordinate axis, Etc. **The deflection:** Plates can be reversed and/or a proton could be used. Plate charge may not be marked, but may be a question. The electric field will not be drawn, instead they will require you to draw the field or to state its direction according to the coordinate axis, Etc.

3–17 Capacitance

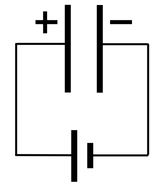


Fig 17.1

Capacitors or Charged Plates: Large Q. Capacitors are a way to ***store electrical charge and energy***. A voltage (potential difference) is needed to create the static electric build up on the plates. The plates can be discharged, releasing energy, by providing a path for the electrons to flow between the plates.

Capacitance: C, is the capacity of the capacitor to hold charge. Think of batteries as electrical pumps, and voltage as electrical pressure that makes it all happen. $\boxed{C = \varepsilon_0 \frac{A}{d}}$. Increasing the area increases the space for the charges on each plate. Since each plate holds like charges, that repel each other, increasing the area creates more room for the charges and lowers electrostatic repulsion. If the distance between the plates is lowered then the positive charge on one plate come closer to the negative on the other. The plates attract each other. To keep charges from jumping from one plate to the other an insulating is placed between the plates. For the AP Physics B class the insulating substance will be a vacuum.

Dielectrics: The real formula for capacitance is $\boxed{C = \frac{\kappa \varepsilon_0 A}{d}}$. ε_0 is the permittivity of free space, and κ is the dielectric constant. The dielectric constant varies from substance to substance. $\kappa = 1$ for a vacuum, resulting in the above equation.

$\boxed{C = \frac{Q}{V}}$ is the capacitance C in terms of the charge stored Q and the batteries potential (pressure) V to pump charges onto the plates. Since the charges would prefer to stay evenly distributed and neutral on both plates work must be done to separate the charges. Electrons, which are loosely bound by their atoms, are made to move from one plate to the other, by the potential V. Remember electrons move in a direction that is opposite that of the electric field. They fall upward toward the positive plate and gain potential energy. $\boxed{U_c = \frac{1}{2} QV = \frac{1}{2} CV^2}$ During this process there is a change in energy, or work.

Disconnect the battery and you have stored the charges on the plates. Connect a wire between them and the charge can flow. The quantity of energy that was stored is released and can be used to do an equal quantity of work. You can think of electricity as water. If you pump water up to a water tower it has high potential energy. If you turn off the pump it will flow back to the ground. A capacitor is an electrical water tower where charge is stored temporarily until it is needed. So if the electrical circuit needs to flush all of its toilets simultaneously the capacitor is allowed to discharge. It is then refilled for the next big flush. Q is the water pumped into the tower. V is the pumps pressure. Why is $U_E = qV$ while $U_c = \frac{1}{2} QV$?

Remember Q is the charge of the plates themselves. The first electron moved onto the plate does so easily. But the second is repelled by the first, and the third by the first two. It becomes harder to put more electrons on the plate since repulsion becomes higher with each electron. The positive plate becomes more positive pulling the electrons back as well. The rate of charges moved starts out high and declines to zero. Average the high rate initially with zero at the end and you get half of the initial rate. If you pump water up a water tower it goes fast at first. But, the weight of the water filling the tower begins to make it harder to pump, until the force of the water in the tower pushing down equals the force of the pump pushing water up. Average the fast pumping at the start with zero pumping at the end, and you get half.

Circuits containing Capacitors: Several capacitors in series or parallel, or in combination can add up to act like one capacitor in a similar manner as resistors do. The rules for capacitors in circuits are ***opposite*** the rules for resistors (presented later). However, similar problem solving techniques apply. $\boxed{\frac{1}{C_s} = \sum_i \frac{1}{C_i}}$ $\boxed{C_P = \sum_i C_i}$

Example 17.1: Equivalent Capacitance

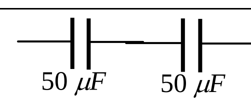


Fig 17.2a

What is the equivalent capacitance of two 50 μC connected in series, Fig 17.2a?

$$\boxed{\frac{1}{C_s} = \sum_i \frac{1}{C_i}} \qquad \frac{1}{C_s} = \frac{1}{50\mu F} + \frac{1}{50\mu F} = \frac{2}{50\mu F} \qquad C_s = \frac{50\mu F}{2} = \boxed{25\mu F}$$

Placing capacitors in series creates less capacitance. Sometimes you need a specific amount of capacitance, but can't locate a capacitor with the exact needed value. The battery has a harder time pumping the two capacitors up, in this configuration.

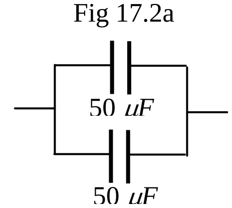


Fig 17.2b

What is the equivalent capacitance of two 50 μC connected in parallel, Fig 17.2b?

$$\boxed{C_P = \sum_i C_i} \qquad C_P = 50\mu F + 50\mu F = \boxed{100\mu F}$$

Placing capacitors in parallel is the same as increasing the area of the plates and therefore increases the capacity of the new combined capacitor

3–18 Current, Resistance, and DC Circuits

Current: The flow of electricity. Current I is considered positive (due to old convention). We now know that the electrons flow, but think of current as positive. To talk about actual electron flow we must say electron current or negative current. Positive current flow follows the direction of the electric field, so negative flow is counter to the electric field.

$\boxed{I = \frac{\Delta Q}{\Delta t}}$ It's a rate (divided by time). But, it is unlike velocity where we measure the distance the car went. Instead we stand still and count how many charges, Q (large amount of charged particles), go by in an amount of time (1 second).

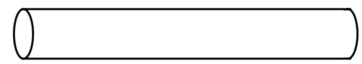


Fig 18.1

Resistance: When water flows down a stream it runs into resistance, such as rocks and sand, etc. When current flows in a length of wire, Fig 18.1, internal properties of the wire slows the current. Resistance is like friction countering the forward progress of the electrons. Conductors slow the current very little, while insulators have lots of resistance, and slow it drastically. All appliances, and even the sources of electricity like batteries have resistance. There are also actual resistors built into circuits to help control electrical flow to exact quantities in various parts of the circuit.

$\boxed{R = \frac{\rho \ell}{Area}}$ Resistance is a function of resistivity, ρ, wire length, ℓ, and cross sectional area. Resistivity, ρ, is like the coefficient of friction. It is derived by experimentation. Different materials have different natural resistances. Gold has very low resistance, while copper is not quite as good, but it is cheaper. So let's worry about length and cross section of wire. Make length small and area big, so the answer is d. The longer the wire the more resistance it has. And resistance is like friction. What type of energy does some of the KE turn into when an object is slowed by friction? Heat. What kind of energy is produced when charges are slowed down by resistance? Heat. What do you feel when you touch an electrical component, like a stereo? It gets hot. Heat loss is disadvantageous. You're losing valuable energy, wasting money on you electrical bill, and increasing entropy. Hot wires also have more resistance. Minimizing resistance is advantageous. But, sometimes you need to create resistance if you have components that can only handle certain amounts of power, energy, voltage, etc. In addition wires themselves have like resistors. One goal in circuit design is to shorten the wires between components to minimize power loss and heat. How much energy is lost in the transmission wires from Hoover Dam?

Important equations for circuits.

$\boxed{V = IR}$ $\boxed{P = IV}$ $\boxed{P = \frac{W}{t}}$ $\boxed{P = I^2R \ \ (not \ \ given)}$ $\boxed{P = \frac{V^2}{R} \ \ (not \ \ given)}$

DC Current: Direct Current is created in batteries, by an electrochemical reaction. These reactions follow entropy and run in one direction producing one way current that is also even in the amount that is produced. The reaction can be reversed, as entropy can, if you put more energy into recharging the battery than you will recover from the battery when it is used. Direct Current, means it travels in one direction only following the electric field lines. The battery pumps charges by creating a potential difference (voltage) between the ends of the circuit. The positive terminal (positive plate) is a region of high potential energy. At the other end of the circuit (wires and components) is a negative terminal (negative plate) that is a region of low potential energy. In a 12 V battery the positive plate is the 12 V plate, while the negative plate is the 0 V plate. Positive current (positive charge) wants to fall toward the ground (negative plate) through the potential difference. This is the direction of the electric field, which is the direction positive charge moves. (Just remember the electrons really flow, so it's all backwards. But, mathematically you get the same numbers).

AC Current: This current comes from generators and is best understood when looking at electromagnetic induction. For now AC current is a current that decelerates to a stop and then accelerates in the opposite direction for a while, then it repeats this process over and over again. The charges move back and forth at 60 Hz (60 times a second) in US circuits. Not only does direction change, but so does the amount of current. The slowing, stopping, and speeding up creates a sinusoidal current over time.

Skier Analogy: A ski lift (battery or generator) elevates skiers (charge) from the lodge (negative plate, 0 V) to the top of 12 V Mountain (positive plate, 12 V) raising both they're potential ($V = Ed$) and potential energy ($U_E = qV$). The skiers (charge) must follow the gravitational field (electric field) back to the lodge (negative plate, 0 V) through the ski resort (circuit). There are several possible runs (wires) for the skiers (charge) to use. Because the skiers (charge) are losing potential energy as they fall, and because energy must be conserved, the potential energy must be turning into another form of energy. It is turning into the kinetic energy of the skiers (charge). Then the skiers (charge) run through gates (appliances, light bulbs, etc.) and they transfer kinetic energy to the gates doing useful work (browning toast, lighting up your room). Unfortunately, the skiers also loose some energy in the gates (appliance, lamp, etc.) to friction (resistance), which creates heat. Also the mountain is covered with moguls (resistors) put on the slopes to purposely slow the skiers (current). As they ski (flow) down the mountain they loose potential energy. They use up all of their potential (height) and potential energy going through gates (appliances, lights, etc.) and moguls (resistors). The skiers (charge) arrive at the lodge with no energy and no potential. They are sent through the process until the switch is opened (resort is closed).

Circuits Containing Resistors

Series: All resistors are in line. Resistors are like sections of wire, designed to slow current. Examine the resistivity of two sections of wire placed in series, as shown in Fig 18.2. $R = \dfrac{\rho \ell}{Area}$

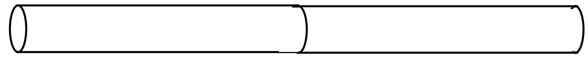


Fig 18.2

ℓ is doubled, but cross sectional area stayed the same. So R doubles. If there were three wires R would triple. This makes sense. Lets pretend they make your commute on the freeway longer, but they didn't build more lanes to widen the road. So you experience the same traffic (resistance) for two or three times as long a road. So in series Resistance adds $\boxed{R_s = \sum_i R_i}$ or $R_s = R_1 + R_2 + R_3 + \ldots$

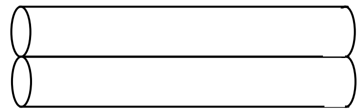


Fig 18.3

Now xamine the resistivity of two sections of wire placed in parallel, shown in Fig 18.3. ℓ stays the same, but cross sectional area doubles. It's more complicated than above. Area is in the denominator. If ***Area*** doubles R is cut in half. If there were three wires R would be cut in third. This makes sense. Its like they expanded the freeway from four lanes to 8 or even 12. You travel the same distance as before but now have less traffic and congestion.

So in parallel $\boxed{\dfrac{1}{R_p} = \sum_i \dfrac{1}{R_i}}$ or $\dfrac{1}{R_p} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3} + \ldots$ **Please don't solve for *1/R_P*. Invert you final answer.**

Current in Circuits: Charge is conserved quantity. If the battery pushes 1 A of charge it is pushing 1 *C* every second into the circuit. Those charges must travel through the rest of the circuit at that rate and must arrive back at the battery at that rate. So in a series circuit, with one path only, the rate that charges move must be the same everywhere. So in a series circuit charge stays the same. But, in a parallel circuit current arrives at a junction and splits up. If 1 A arrives at the junction, then 1 C is arriving every second. Some charges will take one path, while the others will go the other way (ways). But the 1 A splits up, so the current in each section of the parallel circuit must add up to 1 A. You can't have more current in the branches than you had leading up to the branches.

Voltage in Circuits: $V = Ed$. So voltage depends on how far away you are from the battery. If you have a 12 V battery, for example, the positive terminal is 12 V, while the negative terminal is 0 V. As charges move through the circuit they fall away from the positive plate and towards the negative plate. They are farther and farther away from the 12 V plate. Each appliance or resistor uses up some of the voltage. So every time there is resistance voltage is subtracted. The voltage must drop to 0 V when the charges reach the battery, so the resistance in the circuit must use up, subtract all 12 V, in this example. In a series circuit the voltage of the resistors and/or appliances must add to the same value as the battery produces. In a parallel circuit the current takes different paths but, but you travel the same distance from the battery regardless of the path. So in a parallel circuit voltage stays the same.

Series Current stays the same (resistors in line slow traffic in the whole circuit) $I_S = I_1 = I_2 = I_3 = \ldots$
Voltage adds $V_S = V_1 + V_2 + V_3 + \ldots$ (cars are all pushed down a single path)

Parallel Current adds $I_S = I_1 + I_2 + I_3 + \ldots$ (current can choose paths, but the total must split between paths available)
Voltage stays the same $V_S = V_1 = V_2 = V_3 = \ldots$ (cars have an equal pressure down any path)

Ohm' Law: $V = IR$ for devices in electrical circuits.

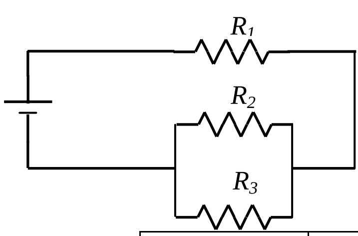


Fig 18.4

Redrawn on the right to match.the skier analogy

	V	I	R
Battery	+21 V	12 A	1.75 Ω
R_1	-12 V	12 A	1 Ω
R_2	-9 V	9 A	1 Ω
R_3	-9 V	3 A	3 Ω

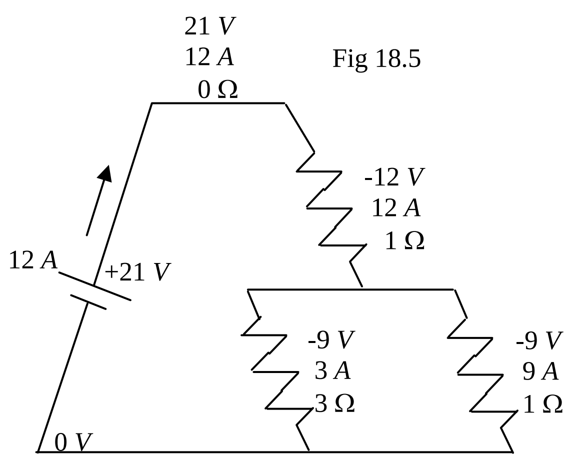


Fig 18.5

<u>Solve the problems on the DC Circuit Worksheet to become familiar with these and other similar computations.</u>

Kirchhoff's Rules: Useful for complicated circuits, where there may be more than one battery, and the directions of the batteries even oppose each other in portions of the circuit. Refer to Fig. 18.4 and 18.5 on the previous page.

1. The sum of the currents entering any junction in a circuit must equal the sum of the currents leaving that junction. $\boxed{\sum I_{in} = \sum I_{out}}$.

   This explains the parallel portion of the preceding circuit. Let's look closer at the parallel junctions in Fig 18.4, isolated and enlarged here in Fig 18.6.

   12 A of current enters the junction at point A. Charge is conserved so all 12 A must leave the junction at point A. The amount going down the two paths is controlled by and proportional to the amount of resistance. The 1 Ω resistor receives 9 A, while the larger 3 Ω prevents current flow to a higher degree, and only receives 3 A.

   The current going through the two branches unites again at point B. The 9 A and 3 A coming into the junction equals the 12 A leaving the junction.

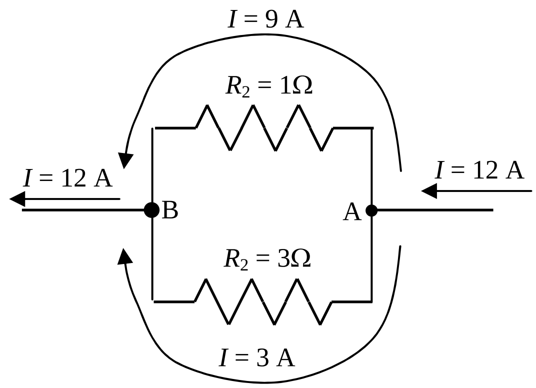


Fig 18.6

2. The sum of the potential differences across all circuit components of any closed loop must be zero. $\boxed{\sum \Delta V = 0}$. Batteries and generators are thought of as pumps that raise the charges to a higher potential. From this higher potential the charges can fall through the circuit and do work (light lights, run microwave ovens, etc.). So, when charges move through a battery the voltage is positive. When they are traveling back down hill, through the components in the circuit, the voltage is negative. If you look at the chart below Fig 18.4, on the previous page, you will see that the signs on the voltages have been included. Added together they total to zero. After all, if the ski lift starts at the lodge and goes up 400 m, the ski runs better go down 400 m to finish there. Otherwise the it will be a very strange resort.

Electromotive Force: ε or *emf*. Not really a force. It is a potential, or voltage, created to make electrons move.

Sources of *emf*: A source of electric field (battery or generator), that makes electrons flow through the circuit. Since *emf* is essentially an electric potential it substitutes into many equations that contain electric potential V (voltage). As an example: under certain circumstances $\boxed{V = IR}$ can be written as $\boxed{\varepsilon = IR}$. Let's see when this is true.

Internal Resistance: *r*. Every device consists of wires that electricity must pass through, so every device has resistance. This is true of sources of *emf* as well (batteries and generators). The resistance of a battery or generator is known as internal resistance, and is denoted with a small r to distinguish it from the resistance of other components (capital R). Let's look at a battery, with internal resistance, in the circuit in Fig 18.7. in greater detail.

The whole battery is drawn as a rectangle, from point A to point B. It consists of some plates where an electrochemical reaction generates ***electromotive force, emf***, $\varepsilon = 6\text{V}$. However, the battery is not perfect. It has an ***internal resistance*** of $r = 1\Omega$. This internal resistance subtracts voltage from the emf. The voltage that is left over is called the ***terminal voltage***. It would be the voltage between points A and B.

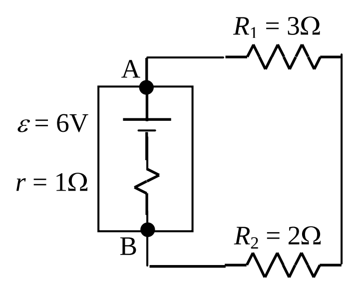


Fig 18.7

Example 18-1: emf, Internal resistance, and terminal voltage.

Treat this like any circuit problem. You can use the rules for current, voltage, and resistance. It is helpful to use a chart, like the one below Fig 18.4, on the previous page. You can also use Kirchhoff's Rules. Just treat the internal resistance like any other circuit component. This means it has negative voltage. This is a series circuit $R_s = \sum R_i = 1 + 2 + 3 = \boxed{6\Omega}$. Plug in *emf* and total R into $V = IR$, $I = \frac{V}{R} = \frac{6V}{6\Omega} = \boxed{1A}$. Since this is a series circuit (including r), $I_S = I_1 = I_2 = I_3 = ...$

	V	I	R
emf	+6 V	1 A	6 Ω
r	-1 V	1 A	1 Ω
R_1	-2 V	1 A	2 Ω
R_2	-3 V	1 A	3 Ω

Use $V = IR$ to complete the each row of the chart. Notice that *emf* was used instead of the battery. The reason is that the battery is composed of both the *emf* and the internal resistance. The terminal voltage of the battery is found using $\boxed{V = \varepsilon - Ir}$.

$$V = 6 - (1)(1) = \boxed{5V}$$

<u>When a problem says that internal resistance is negligible, then it is so close to zero as not to matter mathematically</u>. So, $V = \varepsilon - Ir = \varepsilon - 0$ In this case $\boxed{V = \varepsilon}$, and in these problems V and ε are completely interchangeable.

3–19 *RC* Circuits

Note: *RC* Circuits are needed for AP Physics C: physical science majors, calculus based.

***RC* Circuits:** Circuits containing both resistors and capacitors in series. The circuit in Fig 19.1 is an *RC* circuit, composed of *emf* source ε, resistor R, capacitor C, and switch S. The circuit in Fig 19.2 is a resistor circuit when the switch is in one position and a capacitor circuit when the switch is in the other position. Do not start the math without looking closely at the diagrams. We will focus on the *RC* circuit in Fig 19.1. You already have the skills to solve the other circuit.

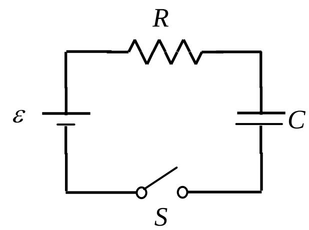


Fig 19.1

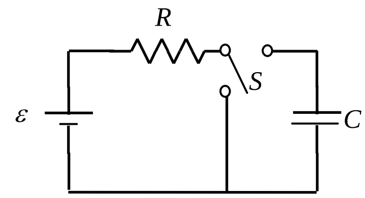


Fig 19.2

Initial conditions: Before the switch is thrown, in Fig 19.1, there is no current in the circuit. The battery produces an *emf*, but the circuit is incomplete. There is no electric field in the circuit and the charges lack a loop to follow. There are no charges built up on the capacitor. There is no resistance in the resistor.

Instantaneous Conditions, Right When Switch is Thrown (time t_0): An electric field traveling at the speed of light extents from the positive terminal of the battery to the negative terminal. The electric field accelerates charges (electrons moving opposite the field, from the negative terminal to the positive terminal. They do not accelerate to the speed of light as they encounter resistance (much like a sky diver hits terminal velocity due to air resistance). Initially there are no charges on the capacitor. This means there are no charges repelling the first charges arriving at the capacitor. So, the full current of these speeding charges move through the resistor. Initially the circuit behaves as though the capacitor does not exit, as shown in Fig 19.3. At this point the formula $V = IR$ applies, if it is adapted to show *emf* and initial current I_0 (at time t_0), as follows, $\varepsilon = I_0 R$. Then rearrange to solve for the initial current. $\boxed{I_0 = \frac{\varepsilon}{R}}$

A more formal approach uses Kirchhoff's 2nd Rule: Potential around a closed loop adds to zero. The potential of the battery minus the resistor and minus the capacitor must equal zero., $V - IR - \frac{Q}{C} = 0$. At this time there is no charge stored on the capacitor, so $Q = 0$. Substituting $\varepsilon - I_0 R - \left(\frac{0}{C}\right) = 0$, and solving $\boxed{I_0 = \frac{\varepsilon}{R}}$ leads to the same answer.

<u>Right now the potential Difference of the battery appears entirely across the resistor.</u>

At some time t later: Once charge begins to flow as a current, it is deposited on the capacitor. The charges are loaded onto the capacitor very easily at first. But, as the charges build up so does the electrostatic repulsion. This process is a logarithmic process. It begins quickly, with a lot of charge being deposited. Then it becomes more and more difficult to fight the electrostatic repulsion and the charging gradually tapers off.

The current in the circuit at some time t is solved using $\boxed{I_t = \frac{\varepsilon}{R} e^{-t/RC}}$. The value RC has special significance. It is called the time constant τ, where $\boxed{\tau = RC}$. It is the time for the current to decrease to $1/e$ of its initial value.

<u>Right now the potential Difference of the battery is split between the resistor and the capacitor.</u>

After a very long time: When a long time has passed the capacitor will become full. As a result no current will flow in the circuit. No current is flowing through the resistor, so it is as thought the resistor does not exist. $C = \frac{Q}{V}$. Rearrange to solve for potential $\boxed{V = \frac{Q}{C}}$. Using Kirchhoff's 2nd Law would give the same result. $V - IR - \frac{Q}{C} = 0$ becomes $\varepsilon - (0)R - \frac{Q}{C} = 0$ which simplifies to $\boxed{V = \frac{Q}{C}}$

<u>Right now the potential Difference of the battery appears entirely across the capacitor.</u>

Capacitor Charge vs. Time Graph: Fig 19.3 shows the charge on the capacitor over time. As discussed before the charge on the capacitor is zero initially. When the current begins to flow the charges initially load onto the plates of the capacitor very quickly. In the beginning there are few charges on the plates that would repel additional charges. But, as the capacitor become more full the electric potential has to push charges against an ever increasing electrostatic repulsion. It becomes harder and harder to load more charge. Eventually the potential difference of the capacitor equals the potential difference of the battery. No more charges can load at this point. The maximum charge on the capacitor at this point is $\boxed{Q = C\varepsilon}$. It is shown as a dashed horizontal line marking the limit of charge on the graph. When the time constant is reached the capacitor is 63.2% charged.

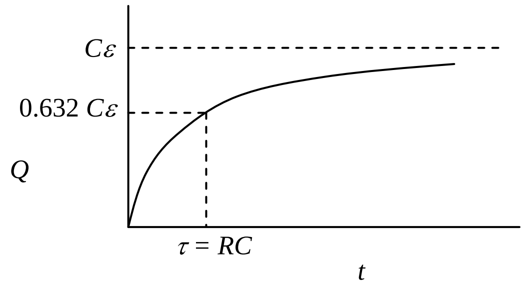


Fig 19.3

Current vs. Time Graph: Fig 19.4 shows the current in the circuit over time. Initially the current flows as though the capacitor is not present, $\boxed{I_0 = \dfrac{\varepsilon}{R}}$. But, as the capacitor charges the current decreases over time, according to $\boxed{I_t = \dfrac{\varepsilon}{R} e^{-t/RC}}$. Eventually when the capacitor reached a full charge the current stops entirely. $\boxed{I_0 = 0}$ charge on the graph. When the time constant is reached the current is at 36.8% of its initial value.

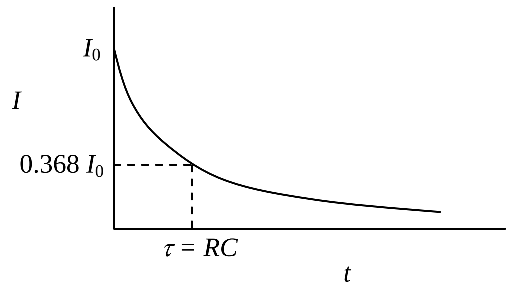


Fig 19.4

Discharging a Capacitor: The battery is removed from the circuit, Fig 19.5. The switch is thrown and the charge stored on one of the plates moves through the circuit and the resistor, toward the other plate. This process continues until the potential difference, originally created by the capacitor, drops to zero. Current flows very quickly at first, and then tapers to zero.

The charge on the capacitor varies by $\boxed{Q_t = Qe^{-t/RC}}$

The current in the circuit varies by $\boxed{I_t = -\dfrac{Q}{RC} e^{-t/RC}}$ and $\dfrac{Q}{RC} = I_0$

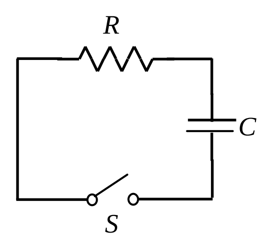


Fig 19.5

3–20 Magnetic Fields

Magnetism: Magnetism is the result of spinning or moving charges. Each electron and proton acts like a tiny magnet. In most substances the protons and electrons balance each other in a manner that cancels the atoms overall magnetism. But in some substances like iron there is an imbalance that is not cancelled. In these substances the atoms act like tiny magnets. If a group of these atoms line up with all their north poles in the same orientation the group or domain acts like a larger magnet. If the smaller domains all line up in a bar of iron, for example, then the entire bar behaves as a magnet, with a north pole and a south pole. It is similar to electricity, where unlike poles attract, and like poles repel. It is different from electricity since, separate poles cannot exist by themselves. Magnetism and Electricity are interconnected. Electric fields can influence magnets, and magnetic fields can influence moving charged particles and current carrying wires.

Magnetic Field Direction In Relation to Fixed Magnets: The magnetic field B comes out of a north pole (like E comes out of + charges) and goes into a south pole (like E goes into – charges). It must leave and enter the poles perpendicular to the surface of the magnet. The magnetic field direction is defined as the direction a north needle of a compass points. The north needle points away from north and towards south. So the Earth's magnetic south pole is located near the Earth's geographic north pole. B is a measure of magnetic field strength, and is analogous to E for electric fields, and g for gravitational fields.

The Magnetic Field Around a Current Carrying Wire (Right Hand Rule, Case 1)

A current carrying wire generates a magnetic field around it. The direction of the field is the direction the fingers curl around the wire if the thumb points in the direction of the positive current. The strength of the field is represented as follows: $\boxed{B = \dfrac{\mu_o}{2\pi}\dfrac{I}{r}}$ This measures the field of a wire carrying a current I at a point in space a distance r from the wire. This formula involves r. Remember that r is used when a field radiates outward from a central source. In electricity radial field are generated by spherical point charges. In gravity radial field are generated by spherical point masses, such as planets. This field radiates out from the wire as shown above right. If you calculate B at point P using the formula above you have also calculated B at any point on the equipotential circle that P lines on. In fact you have calculated B any point on a cylinder surrounding the wire at a radius r.

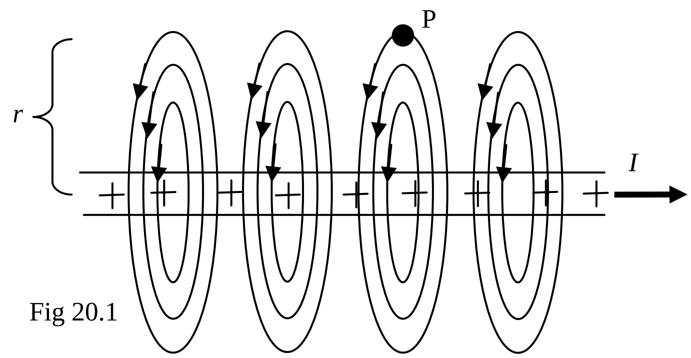


Fig 20.1

What about direction? This is a three dimensional problem, that is diagrammed on paper. You can only diagram what is happening in the plan of the page above and below the wire. We are working in a third dimension and must select a convention for depicting the magnetic field along the z-axis. Since B is a vector we will use an arrow (traditional vector symbol), Fig 19.2. When this arrow is coming at you, you will see the point. So positive z is depicted as dot or circled dot. When this arrow is going away from you, you will see the feathers. So negative z is depicted as an $\times$ or a circled $\times$.

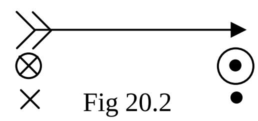


Fig 20.2

Using this convention the field around a carrying wire (Case 1) would look like Fig 19.3.

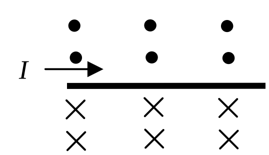


Fig 20.3

Example 6-1: Superposition of Magnetic Field due to Current Carrying Wires

Calculate the magnetic field caused by two crossed current carrying wires at point P in Fig 19.4.

$$B = \frac{\mu_o}{2\pi}\frac{I}{r} = \left(2\times10^{-7}\right)\frac{(2)}{(0.1)} = 4\times10^{-6}T\text{ , +z}$$

$$B = \frac{\mu_o}{2\pi}\frac{I}{r} = \left(2\times10^{-7}\right)\frac{(3)}{(0.05)} = 1.2\times10^{-5}T\text{ , -z}$$

Add vectors: $\left(+4\times10^{-6}\right)+\left(-1.2\times10^{-5}\right) = -8.0\times10^{-6}T$

so it is $\boxed{8.0\times10^{-6}T,\ -z\ direction}$

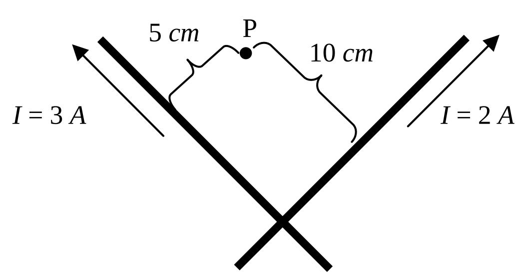


Fig 20.4

The Force of Magnetism on Charged Particle (Right Hand Rule, Case 2)

Charged particles are spinning and generate their own magnetic field and behave like tiny magnets. If you fire them between two large fixed magnets they will be interact with the larger magnetic field. When fields interact there is force. The force is equal and opposite, as always. The fixed magnets attract the particles, just as the particles are attracted to the fixed magnets.

$\boxed{F_B = qvB\sin\theta}$ Force on a charged particle (small q, point charge) traveling (velocity, v) in a magnetic field (B). You can drop $\sin\theta$ if you know the angle is 90^o. ***This force is only felt on the component of velocity perpendicular to the field***. The angle θ is the angle between the particles velocity and the magnetic field that it travels in. In the example to the right the magnetic field is represented by arrows pointing in the $-y$ direction. Particles A and B experience no force since they have a θ equal to 0^o and 180^o. Particle C experiences the most force (if all particles entered the field at the same speed) since $\theta = 90^o$. Particle D will have a force equal to the component of its velocity that is perpendicular to the field. Make sure thatthe problem gives you the correct angle, between the particle velocity and the magnetic field. Problems often try to give the wrong angle to see if you are awake.

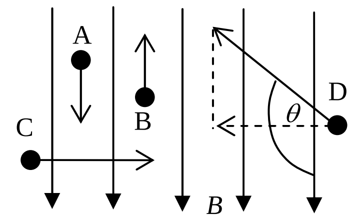


Fig 20.5

$\mathbf{F}_B = q\mathbf{v}\times\mathbf{B}$ is a cross product of vectors. In a cross product the greatest value is obtained when the vectors are perpendicular, allowing $\sin\theta$ to reach its largest value of 1.

Take another look at the problem on the previous page: The field B is going into the page (-z) at point P. If a positive charge $+q$ is at this point and has an instantaneous velocity of $5\text{x}10^4$ *m/s*, what is the magnitude and direction of force on the particle?

$$F_B = qvB = \left(1.6\times10^{-19}\right)\left(5\times10^{4}\right)\left(8.0\times10^{-6}\right) = \boxed{6.4\times10^{-20}\,N,\ +y\ direction}$$

Direction the Particle Will Deflect: Use the right hand rule for positive charge and the left hand for negative charge. The charged particle will follow the right hand rule: The thumb is the charged particles velocity, the extended fingers represent the magnetic field, and the palm slapping represents the force that causes the charged particle to change direction. Remember if a force hits you from behind and parallel to your motion, you speed up. If a force hits you from in front and parallel to your motion you slow down. If a force hits at a right angle to your motion, you change direction (either projectile motion, or circular motion depending on the field). A positive charge is fired into a field in Fig 19.6. The field B extends into the page, -z. Use the right hand rule tracing the path of the particle with the thumb, and keeping the fingers into the board. It traces a semicircle with the palm F_B pointing to the center of the circle.

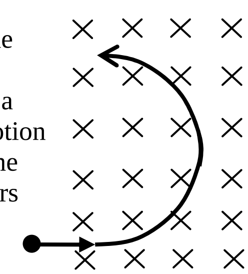


Fig 20.6

$F_C = F_B$ $\qquad \boxed{m\dfrac{v^2}{r} = qvB}$ $\qquad$ we can drop $\sin\theta$ since all vectors are 90^o apart

A charged particle flying in a magnetic field will move in a *<u>circular path</u>*. All vectors are perpendicular to each other. The force on a charged particle in a magnetic field is always perpendicular, so *<u>no work is done</u>* in this case.

Charges Accelerated Through a Potential Difference that Subsequently Enter a Magnetic Field

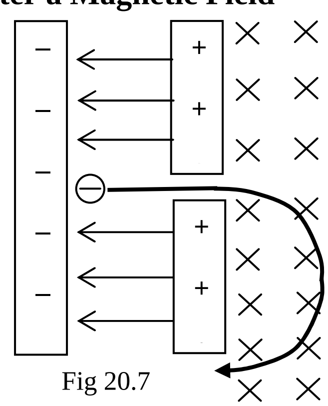


Fig 20.7

$\boxed{qV = \dfrac{1}{2}mv^2\ \ (not\ \ given)}$ In the electric field a charge q is accelerated through a potential difference. It then enters a magnetic field. In this case an electron is accelerated through the potential difference. So you use the left hand to find the direction of the charge once it enters the magnetic field. ***The direction is clockwise***. The force causing the electron into circular motion is F_E. $F_C = F_B$ which leads to $\boxed{m\dfrac{v^2}{r} = qvB\ \ (not\ \ given)}$. Also the electron has a tangential velocity of $\boxed{v = \dfrac{2\pi r}{T}\ \ not\ \ given}$ and a centripetal acceleration of $\boxed{a_c = \dfrac{v^2}{r}}$.

While work is done accelerating the electron (force through a distance) by the electric field, no work is done on the electron while it is in the magnetic field. Substituting 90^o into the work equation $\boxed{W = F\cdot s\cos\theta}$ gives an answer of $\boxed{W = 0}$. At any instant the direction of motion (tangent) is perpendicular to the center seeking force F_B.

What would a proton do? The opposite. In the above scenario the proton would circle counter clockwise. The particle experiencing the motion can be switched, the plates & electric field can be switched. Also the magnetic field could be draw out of the paper. Be prepared for anything. Use the right hand for positive charges and the left for negative charges. Analyze the situation carefully!

The Force of Magnetism on Current Carrying Wire (Right Hand Rule, Case 3)

Note that this is different from case 1. In case 1 we dealt with the magnetic field created by and surrounding a current carrying wire. In this scenario we will put a wire into a magnetic field created by two large fixed magnets. Then we will turn the current on. The wire will then turn into a magnet as well. The magnetic field of the fixed magnets will then interact with the magnetic field of the current carrying wire. When fields interact there is a force. The force will be equal and opposite, but the larger magnetic field is created by fixed magnets. The smaller field will be in a wire which is free to move. So the larger fixed field will cause the wire to jump. This is the basis for the electric motor and for stereo speakers.

$\boxed{F_B = BI\ell\sin\theta}$ is the force of a large fixed magnetic field on a current carrying wire where a length ℓ of the wire is in the larger magnetic field. Use $\sin\theta$ as above.

Direction the Wire Will Deflect: Use the right hand rule for positive current and the left hand for negative current. Use the right hand rule in Fig 19.8. The thumb is the direction of charge movement, as before, except now it is charges Q moving in the wire to create a current I. So the thumb is the direction of positive current I. The fingers are held straight and are the direction of the magnetic field as before. The direction the palm will slap is the direction of the force magnetic F_B. For negative current use the left hand.

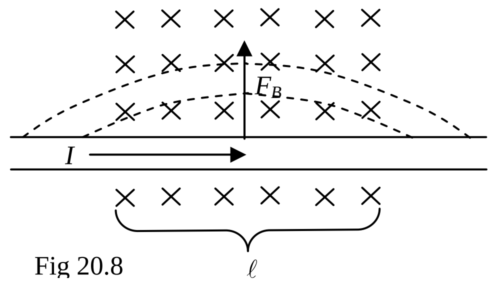


Fig 20.8

Calculus: $\mathbf{F} = \oint I\,d\ell \times \mathbf{B}$

Electric and Magnetic Fields in the Same Location

In some problems a region of space will hold an electric and a magnetic field. Typically the electric plates that generate the electric field E are drawn, while the magnets that create the magnetic field B are located out of and into the page. In this orientation the magnetic field is either in the +z or – z direction, while the electric field is in either the +y or –y direction. But, you can expect other variations of this configuration. Lets follow a charge particle that enters this dual field.

A proton traveling through the charged plates as configured would normally experience projectile motion. The electric field extends downward from the positive plate to the negative plate (direction a positive test charge would move). But, in this problem the proton continues in a straight line. Since the force of the electric field is in the y direction, we should sum our forces in this direction.

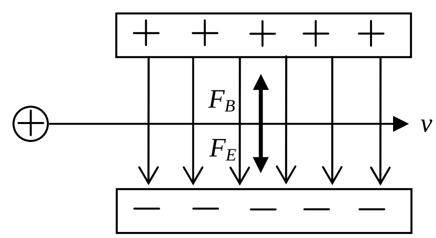


Fig 20.9

$\sum F_y = -F_E + F_B$ The other force that is countering force electric is force magnetic, since this problem deals with two overlapping, but different fields. It is not moving in the y direction so the sum of force must be zero.

$0 = -F_E + F_B$ $\quad \boxed{F_E = F_B}$ $\quad qE = qvB$ $\quad \boxed{E = vB}$

Example 20-1: Combining Some of the Key Problems into One

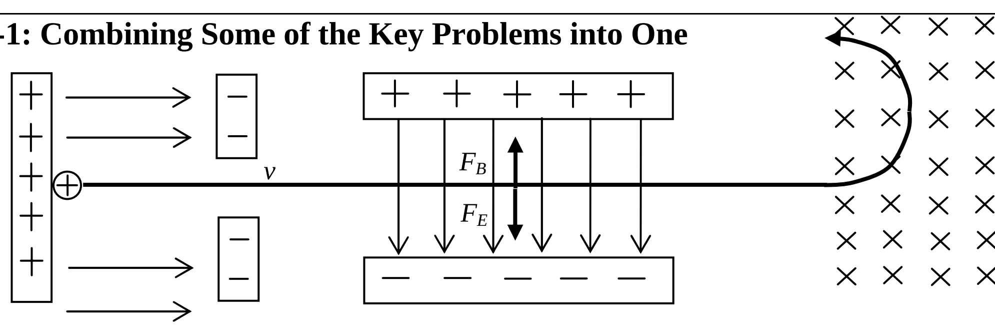


Fig 19.10

$qV = \frac{1}{2}mv^2$

$\boxed{v = \sqrt{\frac{2qV}{m}}}$

***v* is constant for the rest of the problem.**

$F_E = F_B$

$qE = qvB$

$\boxed{E = vB}$ ***B* is in -*z*-direction**

$F_C = F_B$

$m\frac{v^2}{r} = qvB$

$\boxed{B = \frac{mv}{qr}}$

3–21 Ampère's Law

Note: Ampère's Law is needed for AP Physics C: physical science majors, calculus based.

Ampère's Law and The Magnetic Field Around a Current Carrying Wire (Case 1 revisited)

Previously we learned that a current carrying wire generates a magnetic field that circles the wire and varies proportional with the distance from the wire. If we integrate along the path of this circular field, by summing the magnetic field over very small distance it is found to be $\boxed{\oint B \cdot d\ell = \mu_0 I}$. The permeability of free space to the magnetic field is

$\mu_0 = 4\pi \times 10^{-7}\, T \cdot m/A$.

It is useful for calculating the magnetic field around current carrying configurations that have a high degree of symmetry.

Example 21-1: Magnetic Field Around a Current Carrying Wire.

Solve for the magnetic field a distance r from a current carrying wire, in Fig 21.

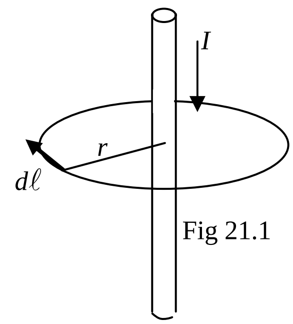


Fig 21.1

Use Ampère's Law, $\boxed{\oint B \cdot d\ell = \mu_0 I}$. As it is highly symmetrical B can be removed from the integral, $B\oint d\ell = \mu_0 I$. The circumference of a circle is known, so integration is not necessary, $B(2\pi r) = \mu_0 I$. Rearrange to solve for B, $\boxed{B = \frac{\mu_0}{2\pi}\frac{I}{r}}$, and we arrive at the algebraic version of the equation, that was given previously.

Solenoid: A solenoid is a wire wound in the form of a helix, as shown in fig 21.2

Magnetic Field of a Solenoid: A The magnetic field in a solenoid is strongest down the middle. The direction follows the right hand rule. The magnetic field is weak around the outside, and in many problems it may say that the field is negligible on the outside. The magnitude of the field is $\boxed{B_S = \mu_0 n I}$

n is the number of turns, N, in the solenoid per unit of length, ℓ. $n = \frac{N}{\ell}$.

A solenoid is a tightly wound coil of wire where each successive coil touches the next coil. This means that the diameter of the wire making up the solenoid is equal to the length of a single turn, $n = \frac{1}{diameter\ of\ wire}$, as shown in Fig 21.2.

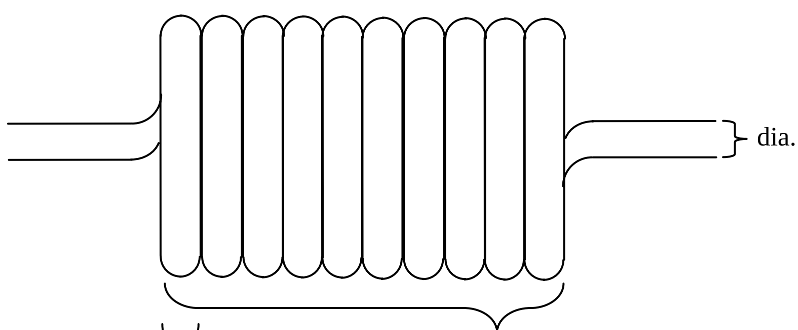


Fig 21.2

Magnetic Flux: The same principles and explanations apply to magnetic flux as they do the electric flux (see previous section on Gauss's Law). $\boxed{\phi_m = \int \mathbf{B} \cdot d\mathbf{A}}$. This is a dot product. The algebraic version is $\boxed{\phi_m = \mathbf{B} \cdot \mathbf{A} = BA\cos\theta}$. ***Caution: remember that flux involves area. When referencing areas angles are measured from a normal (line drawn perpendicular to the surface).***

Gauss's Law of Magnetism: The net magnetic flux through any closed surface is always zero. $\boxed{\oint \mathbf{B} \cdot d\mathbf{A} = 0}$

3–22 Faraday's Law of Induction

Electromagnetic Induction (Right Hand Rule, Case 4)

If a current carrying wire in a larger magnetic field can be made to jump, then could the reverse be true. Could you force a wire to jump into a large magnetic field and then would this in turn cause a current to flow in the wire. This is the basis of electricity production, and is how an electrical generator operates. We must introduce a new quantity known as flux.

Magnetic Flux: $\boxed{\phi_m = B \cdot Area}$ The strength of a magnetic field moving through an area of space, such as a loop of wire. There is an amount of gravity flux through the surface of the earth. There is electrical flux through the space between the plates of a capacitor, and around charged other charged objects. There is magnetic flux around any magnet. In simple terms the flux can be thought of as the amount of field lines passing through a defined area of space. While the field lines aren't really there they are a way of viewing the invisible field. The more lines drawn, and the closer they are to one another the stronger the field and the greater the flux. We will consider the flux through a single simple loop of wire that is located in a magnetic field, Fig.19.9. The wire is full of tiny little magnets (charges). If the wire is forced to move by us (nuclear power, hydroelectric, etc.) these tiny magnets are forced to move in the field. The force of magnetism from the larger surrounding magnetic field, created by a large fixed magnet, will put a force on the tiny charges. As the wire moves the charges move due to the force magnetic, causing a current to flow in the wire. So a stationary loop generates no electricity. Therefore you need a moving loop and as it turns out you need a ***changing flux***. You can change flux several ways: You can hold the magnet stationary and move the wire changing the area (into the field or out of the field). You can rotate the wire in the field, also changing the area. You could hold the wire stationary, and move the magnet, changing the field B's strength as distance changes. You could generate the magnetic field with another current carrying wire, and by changing the current, you also change the field strength.

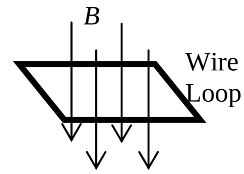


Fig 19.9

Faraday's Law of Induction: $\boxed{\varepsilon_{avg} = -\dfrac{\Delta\phi_m}{\Delta t}}$ Changing flux means the charges in the wire experience a changing magnetic field. An electromotive force is induced. This $\boldsymbol{\varepsilon}$ causes the charges in the wire to move. The charges are trapped in the wire loop and can only travel in a direction consist with both the force and the confines of the wire itself.

Calculus: Faraday's Law is formally written as $\boxed{\varepsilon_{avg} = -\dfrac{d\phi_m}{dt}}$. If the circuit is a coil consisting of N loops then the total induction is given by $\varepsilon_{avg} = -N\dfrac{d\phi_m}{dt}$. In algebraic terms this becomes $\varepsilon_{avg} = -N\dfrac{\Delta\phi_m}{\Delta t}$

Electromotive Force, *emf*: Induction creates electromotive force emf or ε, which really isn't a force, rather it is electric pressure or voltage. However, every device that generates electricity has a resistance, known as ***internal resistance***. So emf is actually the amount of voltage that you could produce if there were no internal resistance in a battery, power supply, generator, or wire loop used to move a current. If internal resistance is included then the equation for induced voltage is $\boxed{V = \varepsilon - Ir}$. Use you're rules for series circuits to deal with the small internal resistance. Treat the internal resistance as a resistor that happens to be next to the battery or power supply. ***In simple problems the internal resistance will be negligible*** $\boxed{Ir = 0}$, similar to a frictionless/airless mechanics problem. Then $\boxed{V = \varepsilon}$.

Lenz's Law: A magnetic field can move charges in a wire by electromagnetic induction. Once the charges start moving they generate their own magnetic field. This new field attempts to counteract the original magnetic field, and is opposite in direction. In order to have opposite direction the charged particles must move in a direction opposite the right hand rule. So in electromagnetic induction the ***minus sign*** in the above equation is a ***reminder that the charges are moving opposite in direction***. **Use the right hand rule as before, but reverse the direction of current.**

Example 22.1: Electromagnetic Induction A solenoid with N turns, length ℓ, and current I is surrounded by a ring of conducting material. **What is the induced *emf* in the ring?** Combine $\boxed{\varepsilon_{avg} = -N\dfrac{d\phi_m}{dt}}$, $\boxed{\phi_m = \int \mathbf{B} \cdot d\mathbf{A}}$, $\boxed{B_S = \mu_0 n I}$, and $\boxed{n = \dfrac{N}{\ell}}$ to produce $\boxed{\varepsilon_{avg} = N\dfrac{\mu_0 \frac{N}{\ell} IA}{t}}$. Simplfy to get $\boxed{\varepsilon_{avg} = \dfrac{\mu_0 N^2 IA}{\ell t}}$

Induction by Moving a Loop Into or Out of a Magnetic Field

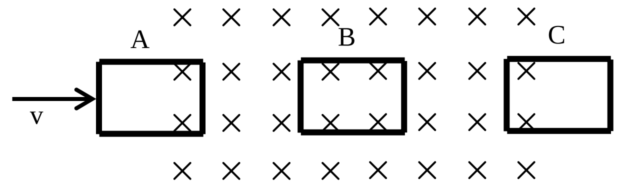


Fig 19.10

The loop, in Fig 19.10, is forced (palm pushing represents the direction the wire is forced) toward the right, with velocity *v*. Look for the part of the loop that is cutting through field lines. This will induce the voltage necessary to move the current. The front edge of the loop is cutting through x's and thus induces a upward current in the leading edge of the loop at location A. The upward motion of charge in the front of the loop causes the rest of the charges in the loop to move as well. The whole motion of charge, induced current is counter clockwise in the entire loop (remember Lenz's Law and the reversal of current direction in induction).

The induced voltage is $\varepsilon_{avg} = \dfrac{\Delta\phi_m}{\Delta t}$ $\qquad \phi = B \cdot Area \qquad$ $\varepsilon_{avg} = \dfrac{B \cdot \Delta Area}{\Delta t}$

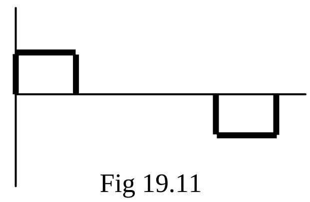


Fig 19.11

The area of the loop in the field is growing, as the loop enters the field, and thus the flux is growing as well. But, the amount of area changing every second is the same since the loop is moving at constant velocity. The front edge of the loop cuts through the same number of field lines every second. ***emf*** does not depend on the size of the area (flux does), rather it depends on the rate of change in flux. This is constant, due to the constant velocity of the loop. Once the entire loop enters the field, position B, a counterclockwise current is induced in the front edge of the loop, while a clockwise current is induced by the back edge. The two opposite currents cancel each other. When the front edge, position C, has left the field, the back of the loop creates a clockwise current. The field is assumed to end at the edge of the x's, or dots for the field drawn out of the page (+z). It is assumed to be a uniform field inside the x's. Graphically the induced emf is shown to the right.

Metal Bar Sliding Across a Loop of Wire

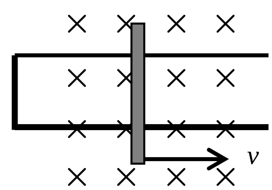


Fig 19.12

If you have a stationary loop of wire placed in a magnetic field, and a metal bar is dragged across the wire loop (Fig 19.12) an electromotive force is produced $\boxed{\varepsilon = B\ell v}$. ***B*** is the magnetic field, ℓ is the length of the bar, and v is the velocity of the bar. This is derived using the formulas above.

$$\varepsilon_{avg} = \frac{\Delta\phi_m}{\Delta t} \qquad \phi = B \cdot Area \qquad \varepsilon_{avg} = \frac{B \cdot \Delta Area}{\Delta t} \qquad \varepsilon_{avg} = \frac{B \cdot \Delta\left(\ell \times d\right)}{\Delta t} \qquad \varepsilon_{avg} = B \cdot \ell \frac{\Delta d}{\Delta t}$$

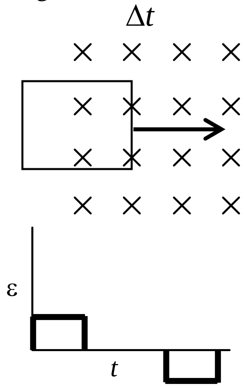
$$\boxed{\varepsilon = B\ell v}$$

Graphing Induced Electromagnetic Force $\boxed{\varepsilon = -\dfrac{\Delta\phi_m}{\Delta t}}$

Remember, current can only be generated by a changing flux. So a closed loop of wire must move through the field, or the loop must be getting larger, or the loop must be rotating, or the magnetic field must be changing.

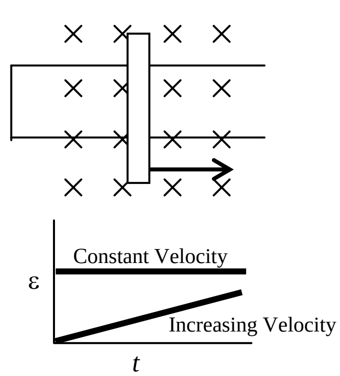
Loop moved thru

$$\varepsilon = \frac{B \cdot \Delta\left(A\right)}{\Delta t}$$



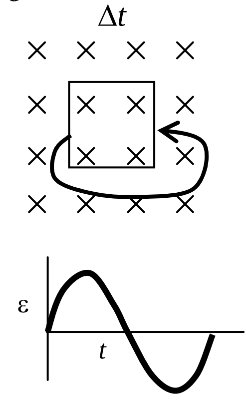
Bar moved, enlarging loop

$$\varepsilon = B\ell v$$



Loop rotates

$$\varepsilon = \frac{B \cdot \Delta\left(A\right)}{\Delta t}$$



3–23 Inductance, *RL*, and *LC* Circuits

Note: Inductance is needed for AP Physics C: physical science majors, calculus based.

Self Induction: When a circuit has a time varying current, an induced *emf* is produced. This induced emf is in the opposite direction of the original current that created it. Consider a simple (loop) circuit consisting of a battery, resistor, and a switch. When the switch is thrown current begins to move. This creates a magnetic field around the wires. The circuit is closed and acts like a single turn loop. The magnetic field induces an *emf* (a second current) in the loop. Due to Lenz's Law this second current counters the original current (reverse direction). The induced current acts to slow the original current that created it. This kind of induced *emf* is known as **self-induced *emf*** or **back *emf***, ε_L.

Inductor: A device intentionally put in a circuit to generate a back emf. A coil of wire generates a stronger magnetic field than a plain wire. As a result, a solenoid used in a circuit acts as an inductor. Using Faraday's Law the formula for inductance is $\varepsilon_L = -N\dfrac{d\phi_m}{dt} = \boxed{-L\dfrac{dI}{dt}}$. L is the inductance (a constant for a specific coil) measured in Henry's (H). The minus sign indicates that the induced *emf* is in the reverse direction of the original current.

RL* Circuits:** A circuit composed of resistors and inductors. We will assume that the wires in the circuit have negligible resistance and inductance. ***Since the back emf opposes the original current, inductor in circuits oppose changes in current in the circuit. If the current wants to rise quickly, an inductor slows the currents rise. If a current wants to stop quickly, an inductor keeps the current going longer.

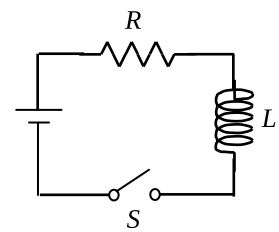


Fig 23.1

Using Kirchhoff's 2nd Law sum all the potential differences of the circuit to zero, $\varepsilon - V_R - \varepsilon_L = 0$. ε is the *emf* (potential) of the battery, V_R is the potential used by the resistor and ε_L is the potential used by the inductor. Substitute, $\varepsilon - IR - L\dfrac{dI}{dt} = 0$. Use this formula to analyze the conditions in the circuit, Fig 23.1, after the switch is thrown. Once charge begins to flow as a current the inductor generates a magnetic field. This magnetic field creates a back *emf*, which opposes and slows the current. The more current through the inductor, the greater the back *emf*, and consequently the more drastically the current is slowed.

Instantaneous Conditions, Right When Switch is Thrown (time t_0): Charges attempt to flow very quickly. Inductors slow this change in current. It is almost as though all the emf is used by the inductor, with no current going to the resistor. (This is the opposite of an RC Circuit, where the current initially goes to the resistor and the capacitor receives none.) Plugging into kirchhoff's 2nd Law, $\varepsilon - (0)R - L\dfrac{dI}{dt} = 0$, and simplifying leads to $\varepsilon = L\dfrac{dI}{dt}$. Since $\varepsilon_L = -L\dfrac{dI}{dt}$, then $\boxed{\varepsilon = \varepsilon_L}$ for just the instant that the switch is thrown.

At Some Time *t* Later: Current at a time t is $\boxed{I_t = \dfrac{\varepsilon}{R}\left(1 - e^{-t/\tau}\right)}$, where the time constant is equal to $\boxed{\tau = \dfrac{L}{R}}$.

After a very long time: Inductors only slow the changes in the circuit, they cannot prevent them. Eventually the current reaches a value that is limited by the resistor in the circuit. Plugging into kirchhoff's 2nd Law, $\varepsilon - IR - (0) = 0$, and simplifying leads to $\varepsilon = IR$. So the current in the circuit after a long time is $\boxed{I = \dfrac{\varepsilon}{R}}$.

Current vs. Time Graph: Fig 23.2 shows the current in the circuit over time. Initially the current flows as though the resistor is not present. The inductor slows the rate of change in the current. Instead of rising to its final value instantaneously the current follows this the equation, $\boxed{I_t = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau}\right)}$. Eventually the current reaches its full value, which is governed by the resistor alone, $\boxed{I = \frac{\varepsilon}{R}}$. At this point it is as though the inductor does not exist. When the time constant is reached the current is at 63.2% of its maximum value.

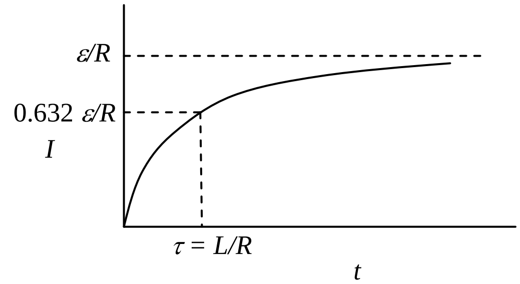


Fig 23.2

Current That is Trying to Stop: Fig 23.3 show a circuit with an additional switch. When switch 1 is closed it operates as above. Current attempts to flow quickly through the circuit, but is slowed in its start up by the inductor. In this case we will allow the current has hit its final maximum value, and then we will throw switch 1 off and switch 2 on. This disconnects the battery. The current should die instantaneously, however inductors slow the change in current. Therefore the current will drop slowly in this scenario. This time the following formula applies, $\boxed{I_t = \frac{\varepsilon}{R} e^{-t/\tau}}$ or $\boxed{I_t = I_0 e^{-t/\tau}}$. The inductor acts to oppose the decrease in current. The graph of this scenario is shown in Fig. 23.4.

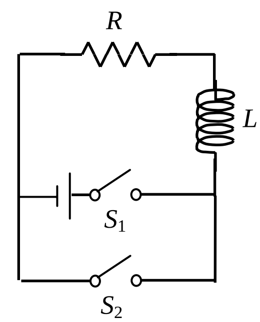


Fig 23.3

Energy in a Magnetic Field: Since the inductor opposes the generation of a current, the battery must do work to create a current. Some of the energy goes into internal energy in the resistor (it gets hotter), but the rest is stored in the magnetic field of the inductor. $\boxed{P = IV}$. Adapt Kirchhoff's 2nd Law, $\varepsilon - IR - L\frac{dI}{dt} = 0$, by multiplying the potentials in it by current to change them into power, $I\varepsilon - I^2R - LI\frac{dI}{dt} = 0$.

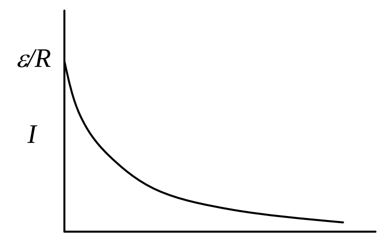


Fig 23.4

Rearrange, $I\varepsilon = I^2R + LI\frac{dI}{dt}$. If we converted these all these terms from power to energy, by multiplying by time, we would have a conservation of energy expression. The energy contained in an inductor is given by the formula $\boxed{U_L = \frac{1}{2}LI^2}$. This is a potential energy, as it is stored and has the potential to do work.

Oscillations in an *LC* Circuit: *LC* circuits are composed of inductors and capacitors. The circuit in Fig 23.5 shows the simplest case. In this scenario the capacitor will be assumed to have a full charge. The switch is thrown and the capacitor attempts to discharge quickly. It acts as a source of *emf*, sending a current through the wires. But the inductor acts to slow the discharge. As the capacitor discharges it looses energy. Where does this energy go? To the inductor. Through conservation of energy $U_C = U_L$ and thus $\boxed{\frac{1}{2}QV^2 = \frac{1}{2}LI^2}$ or $\boxed{\frac{1}{2}CV = \frac{1}{2}LI^2}$. The energy is now stored in a magnetic field around the inductor. The magnetic field then collapses, as there is no current to sustain it. This results in a change in flux in the inductor, which generates a current in the wires. But, this collapse is the opposite direction of the original rising magnetic field from the capacitors discharge. Therefore, the direction of this new induced current is the opposite of the original current created by the capacitor. Now the capacitor begins to charge, but since the current is the reverse, the polarity of the plates is switched. If the top plate were positive at the very start of this scenario, then the top plate is now negative. Once the capacitor is fully charges and all the energy from the inductor has passed back to the capacitor the process begins again, and again… (It is exactly like a spring that oscillated above and below the equilibrium position. Sometimes it is +x and sometime -x. The energy just keeps changing from potential to kinetic.)

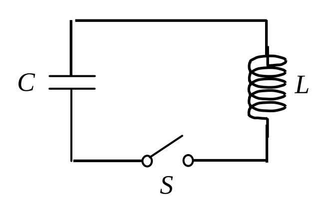


Fig 23.5

3-99 is not a real chapter. This is some hooligan ex-student who thinks it's funny to put his name in the review. He is correct. It is hilarious. Love you Jeff Rose!

-Nico

Unit 4 Wave and Optics

4–24 Waves

Waves Terminology: Waves transport **Energy**, *E*. V**ibrating / oscillating** objects create waves. Waves must travel in a **medium**, except electromagnetic waves, which can travel in a vacuum. **Frequency,** *f*, is the number of vibrations/oscillations, cycles, or revolutions each second. **Period**, *T*, is the time for one complete vibration/oscillation. **Wavelength**, λ, is the length of a single wave, measured from a point on one wave to the exact same point on the next wave. Period is also the time that it takes one wavelength to pass. **Amplitude**, *A*, is the maximum displacement from the equilibrium position (not the top to bottom distance). The positive amplitude peaks are known as **crests**, while the negative regions are known as **troughs**.

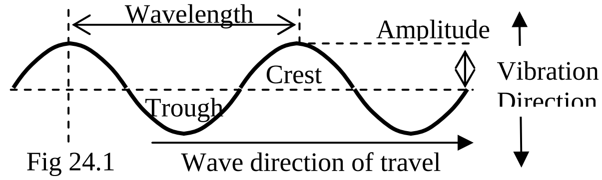


Fig 24.1

Transverse waves: Look like the traditional sin waves, shown in Fig 24.1. The vibrating particles, that cause the wave, oscillate in a perpendicular to the wave direction.

Longitudinal Waves: Compression, or **Shock Waves** where particles vibrate in a direction parallel to wave direction. Longitudinal waves have **compressions** and **rarefactions**, similar to crests and troughs. All waves have the above characteristics, and all waves can be diagrammed as a **Sinusoidal Function**. A **pulse** is a single wave. A **continuous wave** is a series of equal pulses.

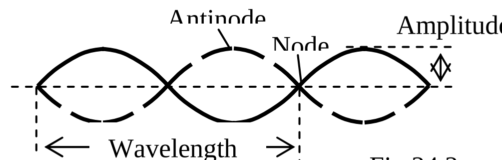


Fig 24.2

Standing wave: Created when a continuous wave strikes a barrier and reflects back upon itself. <u>*Any wave hitting a **hard boundary** reflects with a **180^o phase change***</u>. **In phase waves** have the same wavelength and speed. Their crests line up with each other. They are in step with each other. A 180^o phase change means that the crests of one wave are lined up with the troughs of another. The reflected wave collides with the next incoming wave and superimpose, creating a standing wave. **Nodes** do not move at all, while **antinodes** are points of maximum amplitude. A wavelength is composed of two antinodes.

<u>Wave speed depends on the medium</u>: $\boxed{v = f\lambda}$ depends on the elasticity of the medium. Sound travels faster in metal than in water and faster in water than in air. Light is unusual, it is fastest in a vacuum, slows slightly in air, and moves slowest in denser mediums. <u>***As long as the medium does not change wave speed is constant***</u>. If speed is constant and frequency doubles, what happens to wavelength? It is cut in half. Frequency and wavelength have an inverse relationship. The equation $\boxed{v = d/t}$ also applies to sound and light waves. You can time the distance to lightening by counting the seconds between flash and thunder. When timing sound that makes a round trip (echo or sonar), divide the final answer by 2.

Interference: When two or more waves meet, their amplitudes add.

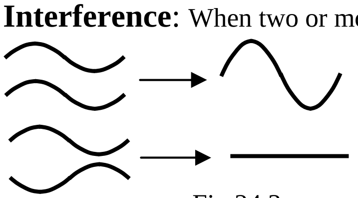


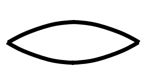
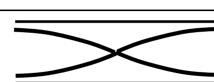
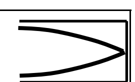
Fig 24.3

Constructive Interference: If the waves are in phase they will add constructively creating a wave with larger amplitude.

Destructive Interference: If the waves are out of phase they will add destructively creating a wave with smaller or even zero amplitude. The waves shown to the left have the same amplitude and wavelength. But, two waves could have different speeds, wavelengths, amplitudes, or phases and add with any combination of these. Therefore a host of complex new wave forms can be generated. Music is one such example.

Sound: The speed of sound in air at 25^o C is 343 m/s (often rounded to 340 m/s). The speed of sound changes with temperature since the density and elasticity of air change as temperatures fluctuate. **Pitch** is frequency and **loudness** is amplitude

Resonance: Natural frequency of vibration, and the basis of musical instruments. Resonance is employed to create louder and richer sounds from string and tube musical devices. We will only consider the mathematical application of the fundamental resonant frequency, the frequency associated with the smallest possible fraction of a wavelength that will fit in a string or tube. Multiples of the fundamental frequencies wavelength would also work, and are known as overtones. We will not work with the overtones.

Strings  ½ wavelength	**Open Tubes**  ½ wavelength	**Closed Tubes**  ¼ wavelength
$v = f\lambda$ and $\lambda = 2L$ $\boxed{v = f2L}$	$v = f\lambda$ and $\lambda = 2L$ $\boxed{v = f2L}$	$v = f\lambda$ and $\lambda = 4L$ $\boxed{v = f4L}$

Electromagnetic Radiation (Light): Travels in packets of energy, called **Photons**. A vibrating charge, surrounded by both an electric and a magnetic field creates electromagnetic waves. Once created the oscillating electric and magnetic field, which are perpendicular to each other, can self sustain each other and as a result can travel through the vacuum of empty space. We see only visible light, which is a very small portion of the entire electromagnetic spectrum. The entire spectrum from weakest to strongest is:

Radio Waves	Microwaves	Infrared (IR)	Visible	Ultraviolet (UV)	X-Rays	Gamma Rays

Low energy, low frequency, long wavelength — High energy, high frequency, short wavelength

Visible light follows ***ROY G BIV*** from low energy to high energy. ***So Blue light has higher energy than red light***.

To see light, the waves must be aimed straight into our eyes to excite the photoreceptors at the back of our eyes. The colors we see are the waves of light that reflected off of an object. The colors that are not present are the ones that passed through the object or were absorbed by the object. Grass absorbs red and blue light needed for photosynthesis, and it reflects the unnecessary green wavelengths.

Law of Reflection: $\boxed{\theta_i = \theta_r}$. All optical angles are measured in relation to a normal line. A normal is a line perpendicular to the surface (Force Normal is perpendicular to the surface). When a surface is curved the normal is perpendicular to a tangent to the curved surface. It is diagrammed as a dashed line.

Ray, smooth surface — Parallel rays, smooth surface — Rough surface, ***Diffuse*** reflection

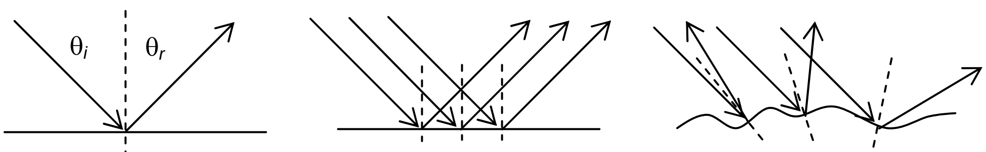


Fig 24.4

On a smooth surface such as a mirror the incident parallel rays of light reflect parallel, and the image formed is identical to the object. But on a rough surface the incident parallel rays are sent in all directions and the image will be **diffuse** (fuzzy).

Transmitting Light: *EM* waves not only travel in vacuums, but that is where they are fastest. The speed of light in a vacuum, c, is $\boxed{c = 3\times10^8\ m/s}$. The ***Index of refraction***, $\boxed{n = \frac{c}{v}}$ was devised as a comparison value. It compares the speed of light in a medium a the known speed in a vacuum. The index of refraction for light in a vacuum is 1.00, and the index can never be less than one. The index of refraction for air is 1.0003, which rounds to $\boxed{n_{air} = 1.00}$.

Refraction: The bending of light as it changes mediums. The ***speed changes, but the frequency does not***, $\boxed{v = f\lambda}$ ***so the wavelength changes*** causing the path of the light to bend. ***Less to more, bends toward***.

Less dense to more dense	Light slows	Frequency same	Wavelength shortens	Bends toward normal
More dense to less dense	Light faster	Frequency same	Wavelength lengthens	Bends away from normal

Snell's Law: $\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$ Given the speed in one or both mediums, the indices of refraction or angles can be determined. With three pieces of information the fourth can be determined mathematically. Fig 24.5 shows light moving from a less dense medium to a more dense medium. n_1 and θ_1 go with the incident medium while n_2 and θ_2 go with the refracted medium. Blue light with its higher energy, higher frequency, and lower wavelength bends more than red light. It will bend more toward the normal (less to more), and it will bend more away from the normal as well (more to less).

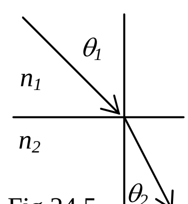


Fig 24.5

Total Internal Reflection: A special case of Snell's Law. If the incident angle is of a certain size it will result in a 90^o angle of refraction. This incident angle is called the critical angle $\boxed{\theta_c}$. At incident angles equal to or larger than the critical angle the light reflects back into the substance. So the light at the critical angle or greater is totally internally reflected. Medium 1 contains the incident ray so θ_1 is the critical angle and θ_2 is 90^o. This only works when you go from a dense to less dense medium.

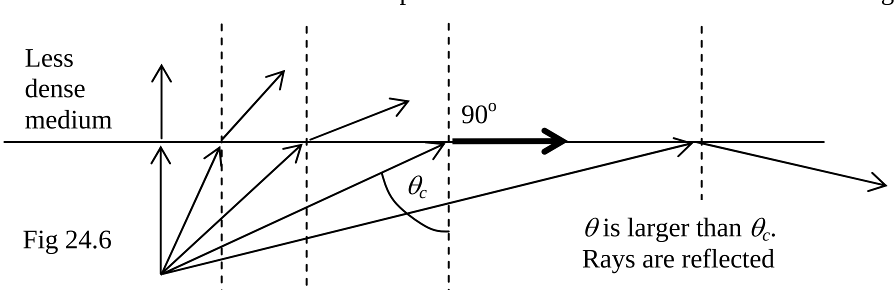


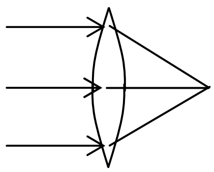
Fig 24.6

$n_1 \sin\theta_c = n_2 \sin 90^o$ $\qquad$ $n_1 \sin\theta_c = n_2$ $\qquad$ $\boxed{\sin\theta_c = \frac{n_2}{n_1}}$

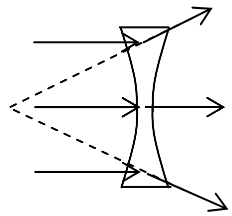
4–25 Geometric Optics

Lenses and Mirrors: Mirrors reflect light and lenses transmit light. They both fall into two main categories. **Converging** lenses and mirrors converge parallel rays of light on the focal point, and thus the focus is $+f$. **Diverging** lenses and mirrors diverge parallel rays of light away from the focus, with a $-f$. The shapes, convex and concave are secondary, but must be memorized.

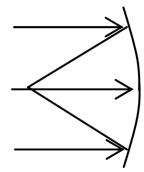
Converging Lense, +*f*
Convex Lense



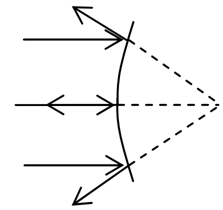
Diverging Lense, -*f*
Concave Lense



Converging Mirror, + *f*
Concave Mirror



Diverging Mirror, - *f*
Convex Mirror



Spherical Lenses and Mirrors: We are working with lenses & mirrors that have been cut from spheres. Each curved surface has a center of curvature. The focal point is located half way between the center of curvature and the lens or mirrors surface. Focal distance is measured from lens or mirror surface to the focal point. $\boxed{F = r/2}$ and the distance to the center is $\boxed{C = 2F}$.

Ray Tracing: Remember light goes through the lens, while it bounces off mirrors.

Rules for Lenses: Rays arriving parallel to the optical axis, either converge on the far focus or diverge from the near focus.

Rays that go through the center of the lens keep going straight.

Rules for mirrors: Rays arriving parallel to the optical axis, either converge on the near focus or diverge from the far focus.

Rays arriving through the focus, go out parallel.
Rays that go through the center of curvature bounce ($C = 2F$) bounce straight back.

Images: Two types of images are formed by light interacting with lenses and mirrors. To find the location and the size of an image use ray tracing practiced in the following worksheet. If the ***forward ray traces*** touch (converge to create an image) the image is considered **real** and has a positive distance from the lens, $+s_i$. ***Real images can be projected on a screen***. ***Real images are always inverted***. But, when rays diverge (separate) the forward ray traces will not intersect. You then must draw a **back ray trace** through the other focus. In this case the image is **virtual**, and since it resulted from the intersection of the negative ray traces it has a negative distance, $-s_i$. ***Virtual images are always upright.***

Converging Lens / Mirror: Object outside of focus	Converging Lens / Mirror: Object at focus	Converging Lens / Mirror: Object inside of focus	Diverging lens / Mirror
Real Images, $+s_i$	No Image: $s_i = \infty$	Virtual Image, $-s_i$	
Inverted, $-h_i$, and $-M$	$f = s_o$	Upright, $+h_i$, and $+M$	

Geometric Optics

$\boxed{f = \frac{r}{2}}$ The center of curvature is located at $2f$. So the focal point is half of the radius. $\boxed{\frac{1}{f} = \frac{1}{s_o} + \frac{1}{s_i}}$ Shows the geometric relationship between the focal length, object distance, and image distance. $\boxed{M = \frac{h_i}{h_o} = -\frac{s_i}{s_o}}$ Relates the magnification to the height of the object and image, and the distance to each. The only conversions needed are in cases where you lack unit agreement. Variables are either all the same or they cancel out. Centimeters are commonly used in optics. The magnification formula tells the relationship between s_i and h_i. These variables always have the opposite sign. Remember negative M does not mean the image is smaller. It means the image is upside down. 0.5x is a smaller image and upright, while –2.0x is a larger inverted image.

For example problems complete the Geometric Optics Worksheet.

4–26 Wave Nature of Light

Diffraction: When light hits the edge of a barrier, as shown in the left diagram in Fig 26.1, it will bend around it. If the barrier is small compared to the wavelength of light the light will pass the barrier uninterrupted. Like water flowing around a buoy. But, if the barrier is large compared to the wavelength of light the waves will bend around the edge of the barrier in a circular fashion. Imagine water waves hitting the end of a jetty, or going through a hole in a jetty.

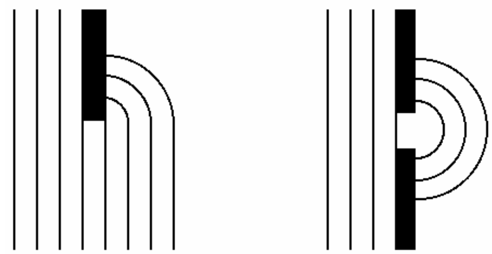


Fig 26.1

Slit Width: The smaller the opening the more pronounced the circular effect, as shown in the right diagram in Fig 26.1.

Huygen's Principle: Every point on a wave front can be considered as a source of tiny wavelets that spread out in the forward direction at the speed of the wave itself. The wave front is formed by the constructive interference of the circular wavelets, as shown in Fig 26.2.

So, if the barrier has a hole in it, the particles of the wave create new circular wave fronts when they exit on the other side. This is why you get the circular patterns shown on the previous page.

The particles of light that create this diffraction pattern are the photons. Each photon generates circular wave fronts that constructively interfere to generate linear wave fronts.

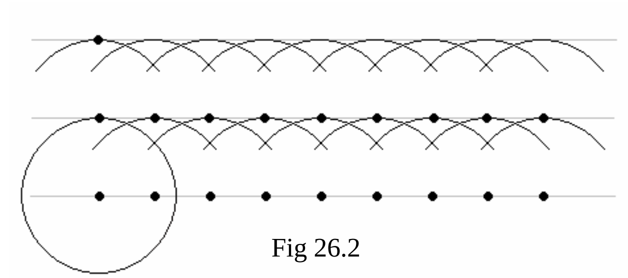


Fig 26.2

Young's Double Slit Experiment: Young shined a monochromatic light source on two very narrow slits. He used a monochromatic light source, since white light consisting of all the colors would result in a rainbow refraction. Light has the same characteristic pattern as water waves and is ***<u>evidence that light behaves as a wave</u>***. Light from two slits interferes with each other. As a result you get dark and light bands if the pattern is shown on a screen.

Interference Fringes: The bands of light and dark that appear at regular intervals, as shown in Fig 26.3. They follow a

m geometric relationship described by the following equations. $\boxed{d\sin\theta = m\lambda}$ $\boxed{x_m \approx \dfrac{m\lambda L}{d}}$

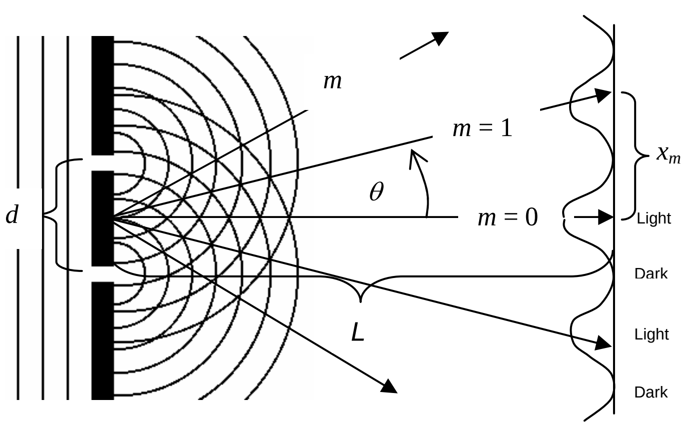


Fig 26.3

d is the distance between the slits.

m is the fringe number. $m = 0$ is the central maximum. It is also the ***<u>path difference in wavelengths</u>***.

Whole numbers are used for the bright ***Constructive Interference Fringes***. $m = 0.0, 1.0, 2.0, 3.0$, etc.

Half numbers are used for the dark ***destructive interference fringes***. $m = 0.5, 1.5, 2.5, 3.5$, etc.

θ is the angle from the midline (from the middle fringe).

L is the distance from slits to the screen.

x_m is the distance from the midline (center fringe) to the fringe being measured.

The second formula $\boxed{x_m \approx \dfrac{m\lambda L}{d}}$ is only approximately equal. If $\sin\theta = \dfrac{x_m}{L}$, then $d\sin\theta = m\lambda$ would become

$d\dfrac{x_m}{L} = m\lambda$. This would simplify to $\boxed{x_m = \dfrac{m\lambda L}{d}}$. But $\sin\theta = \dfrac{x_m}{L}$ is not true. x_m is the opposite and L is the adjacent.

So $\tan\theta = \dfrac{x_m}{L}$ is the true equality. But, as long as the angle is small $\sin\theta \approx \tan\theta$, and then we can use $\boxed{x_m \approx \dfrac{m\lambda L}{d}}$.

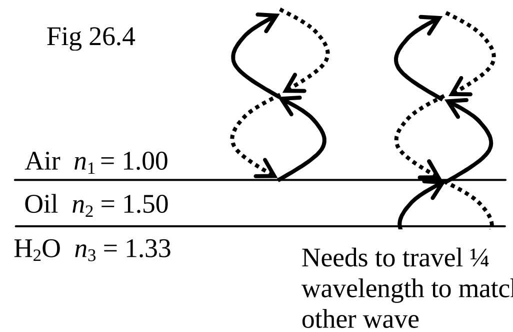
Thin Films: Most people are familiar with the rainbow effect seen when a thin oil/soap film is floating on top of water. The phenomenon can also be seen in the thin film coatings that give modern sunglasses their protective properties. The ***thickness*** of a thin film and the ***density*** of the film and the mediums around it, combine to create the thin film optical effect.

Important background information to keep in mind.

- Denser mediums have higher indices of refraction.
- A reflected ray will change phase by 180^o when going from a medium with a low n to a medium with a high n.
- A reflected ray going from a medium with a higher n to a medium with a lower n is a soft boundary reflection.

Case 1: The middle medium has the greatest density.

Example: Oil Suspended On Water: Light moving from air to oil and then to water. It goes from low density (low n) to a high density (high n), and back to a lower density (lower n). Two beams of light shown as dashed lines strike the thin film, in Fig 26.4. One reflects off the air-oil boundary. The high to low index of refraction change is like hitting a hard boundary. This causes a 180^o shift in the outbound reflected light wave, as seen on the left in Fig 26.4. This is similar to the wave reflection in a guitar string that is tied down, creating a hard boundary. The right light wave makes it through the first surface and reflects off the second surface. The change from oil to water is from more to less dense and the index of refraction gets smaller. This is like hitting a soft boundary, such as air reflecting in an open tube. If the thin film has an exact thickness the two outbound light waves will be in phase. Waves in phase constructively interfere creating a larger wave. In the case of light it creates a brighter light. If the wavelength is an observable color our eye would see a brightening of that color. In the diagram above this occurs when the film is ¼ of a wavelength.



Problem: The oil film is 1x10^{-7} m (100 nm) thick, $n_1 = 1.00$, $n_2 = 1.50$, $n_3 = 1.33$. What wavelengths will results in maximum intensity? $\boxed{thickness_{film} = \frac{1}{4}\lambda_{film}}$ $\lambda_{film} = 4\left(thickness_{film}\right) = 4\left(1\times10^{-7}\right) = 4\times10^{-7}m = \boxed{400nm}$

Caution: You have now found the wavelength in the film (the oil). Your eye will see the color in the air, so this wavelength needs to be converted from oil to air. There is a formula that converts wavelengths based on the index of refraction $\boxed{n_1\lambda_1 = n_2\lambda_2}$. Rewritten to match this situation: $\boxed{n_{air}\lambda_{air} = n_{film}\lambda_{film}}$ $\lambda_{air} = \frac{n_{film}\lambda_{film}}{n_{air}} = \frac{(1.5)(400nm)}{(1.0)} = \boxed{600nm}$.

Is there any destructive interference? Yes, if the thickness of the film is changed to ½ of a wavelength

Conclusion: When $n_1 < n_2 > n_3$

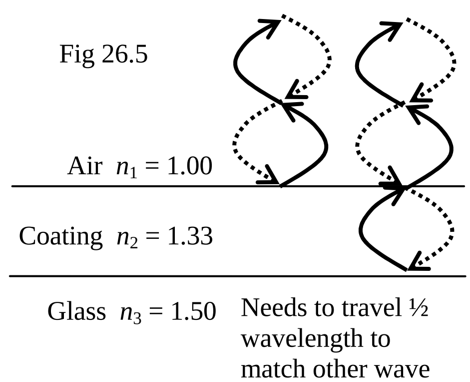
Constructive Interference: $\lambda_{film} = 4\left(thickness_{film}\right)$

Destructive Interference: $\lambda_{film} = 2\left(thickness_{film}\right)$

Followed by $\boxed{n_{air}\lambda_{air} = n_{film}\lambda_{film}}$ $\lambda_{air} = \frac{n_{film}\lambda_{film}}{n_{air}}$

Case 2: Mediums With Progressively Higher Densities

At both boundaries the change is to a higher density (higher n), so there are two 180^o reflections. Here the film needs to be the thickness of ½ wavelength to get maximum intensity. $\boxed{thickness_{film} = \frac{1}{2}\lambda_{film}}$ It is the reverse of the above scenario. Even multiples of ¼ wavelength create constructive interference, while odd multiples of ¼ wavelength create destructive interference.



Problem: The sunglass coating 2x10^{-7} m thick, $n_1 = 1.00$, $n_2 = 1.33$, and $n_3 = 1.50$. What wavelength will result in maximum intensity?

$\lambda_{film} = 2\left(thickness_{film}\right) = 2\left(2\times10^{-7}\right) = 4\times10^{-7}m = 400nm$

$\boxed{n_{air}\lambda_{air} = n_{film}\lambda_{film}}$ $\lambda_{air} = \frac{n_{film}\lambda_{film}}{n_{air}} = \frac{(1.33)(400nm)}{(1.0)} = \boxed{532nm}$

Conclusion: When $n_1 < n_2 < n_3$

Constructive Interference: $\lambda_{film} = 2\left(thickness_{film}\right)$

Destructive Interference: $\lambda_{film} = 4\left(thickness_{film}\right)$

Followed by $\boxed{n_{air}\lambda_{air} = n_{film}\lambda_{film}}$ $\lambda_{air} = \frac{n_{film}\lambda_{film}}{n_{air}}$

Unit 5 Modern Physics

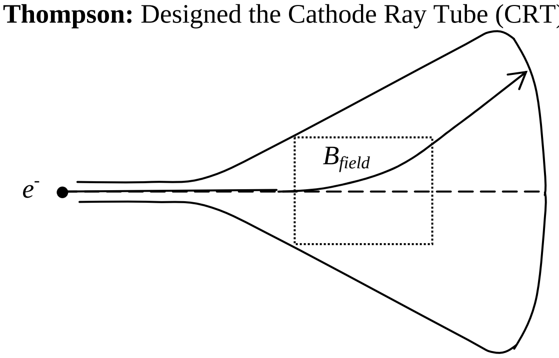
5–27 Atoms, Energy Levels, and Photoelectric Effect

Electromagnetic Radiation: In the previous review sheet we discussed the electromagnetic spectrum.

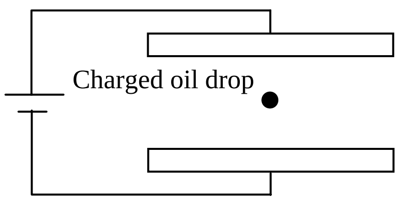
Maxwell: Derived a famous series of four equations that could completely describe electromagnetic effects. His equations, which were a compilation of the work of many other famous scientists, showed the existence and predicted the speed of these waves. However, you won't need the equations now. You can enjoy this chapter of physics in college.

Atomic Theory

Thompson: Designed the Cathode Ray Tube (CRT), the basis for TV and computer monitors. In the partially evacuated tube he had two electrical terminals, thus creating an electric field. When a potential difference was created between the electrodes (plates) he witnessed an eerie beam that came from the negative (cathode) plate and went to the positive plate. Creating a hole in the positive plate allowed the beam to pass through and strike the end of the tube. He surmised the existence of negative particles that must have come from inside the atom. The stream of particles could also be bent by an intervening magnetic or electric field. The atom was no longer recognized as the smallest entity in the universe. The particles making up the beam were called electrons.



Milikan Oil Drop: He charged oil drops with the newly discovered negative particles. He then sprayed the oil drops between two charged plates. He could adjust the potential difference between the plates until the oil drops were suspended in mid air. This means that the force up = the force down. So the electric force up = the gravity force down. $Eq = mg$ so $\boxed{q = \frac{mg}{E}}$ By finding the mass of the oil drop and noting the strength of the electric field he could calculate the excess charge on the oil drop. He performed the experiment many times. He noted that the charge was always a multiple of 1.6×10^{-19} C and he never found a value smaller than this. So he concluded that this is the charge on the electron. After all charge comes in quanta, or whole number quantities. There are no half electrons.



Rutherford: Fired alpha particles at gold foil. Around the foil he placed a screen sensitive to alpha particles. He thought the heavy and fast alpha particle would fire right through the foil. While most did, he did note that many rebounded from the foil. He likened this to shooting a cannon at tissue paper and having the shell bounce back. He postulated that the atom was mainly empty space, but that there was a small dense nucleus comprised of positive particles (protons) located at the center. The empty space would account for most alpha particles passing through. The positive nucleus would explain how the positive alpha particles were bounced back. He did calculations based on the various trajectories of the alpha particles, and he was able to predict the relatively small size of the nucleus. His model of the atom was similar to the solar system (planetary model) with the nucleus (sun) at the center and the electrons (planets) orbiting at a great distance.

Einstein: Postulated that light has a particle nature, and travels in packets of energy known as photons.

Planck: Found the energy of a photon. $\boxed{E = hf}$. The energy of a photon is Planck's constant (*h*) x frequency.

Emission Spectrum: It was noted that when gaseous elements were placed in a tube at near vacuum and a potential difference was placed at the ends of the tube (Thompson *CRT*) different colors were seen. When shot through a diffraction grating the colors showed up as discrete lines with dark areas in between. Formulas were worked out for Hydrogen, the simplest element and the placement of the lines fit a mathematical pattern.

Bohr: He predicted that the atom was similar to Rutherford's model, but Bohr added the concept of energy levels. The fact that atoms only emitted certain frequencies of light implied that the electrons could only occupy certain discrete energy levels. And these electrons could never be in between these energy levels. When photons strike an atom, the electrons (normally in the ground state) of the atom absorb the energy. These electrons (excited) now have higher energy and thus move to higher energy levels within the atom. When the electrons returned to the ground state photons are emitted since the electrons loose energy. But the electrons may drop to a variety of levels on the way back to the ground state. This explains the many colored lines in the emission spectrum and the calculations matched those of the emission spectrum. The **Bohr Model of the atom** is outdated & incomplete, but it is still used to visualize the atom.

Debroglie: Felt that since light can act as both a particle (photon) and a wave, perhaps electrons (a particle) can have a wave property as well. He calculated the wavelengths of electrons $\boxed{\lambda = h/p}$ and found that theoretically an electron in the lowest energy level has one electron wavelength in a ***standing matter wave***. An electron in the second energy level completes two wavelengths, etc. This resulted in the ***Wave Mechanical Model of the Atom***. So the electrons were not jumping from energy levels as Bohr had surmised, rather they were changing wavelength when bombarded by photons.

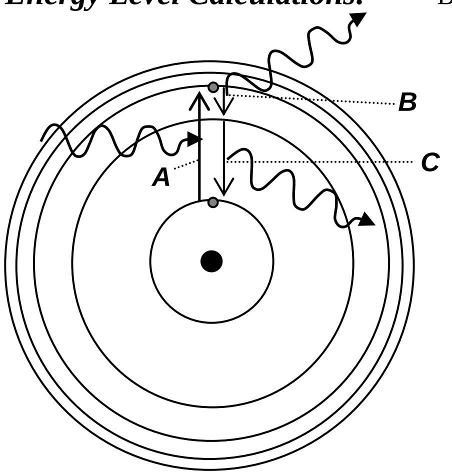
Heisenberg: Postulated that you can never know both the location and the momentum of an electron simultaneously. His ***Uncertainty Principle*** states that you cannot find an electron or predict what it will do, since if you shoot photons at electrons to see what their doing the photons interact with the electrons changing their location and / or their speed.

Schrodinger: Performed the calculations to narrow down the possible locations that electrons may be found in an atom. These statistical shells that electrons occupy are the basis of the current ***Electron Shell Model of the Atom***.

Einstein: Found the mass energy equivalence $\boxed{\Delta E = (\Delta m)c^2}$. Even though a photon is really mass-less energy it does have a mass equivalence. So if its energy can be converted to a mass value, then the photon can be given a mathematical momentum. So Planck's equation is updated. $\boxed{E = hf = pc}$

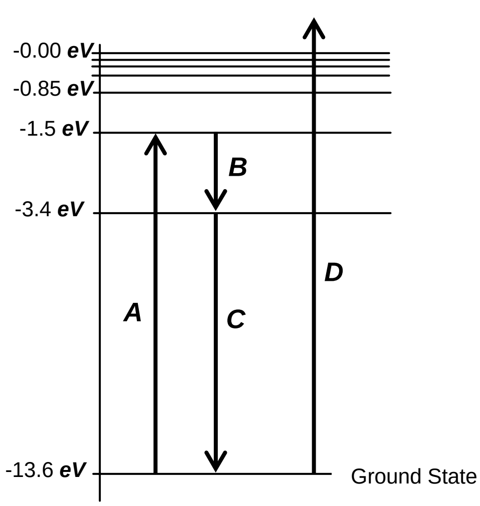
Energy Level Calculations:

Bohr's model has been updated, but the energy levels still hold mathematically.



If a photon of the right frequency and energy were to strike the lone electron in hydrogen while it is in the ground state, the electron would then acquire all of the photons energy. The ***photon is absorbed by the atom***. The energy of the photon determines how much energy the electron receives and thus determines the energy level that the electron is boosted to. Perhaps the electron would receive enough energy to be excited to the 3rd energy level as shown in process ***A***. This excited electron would be unstable and would eventually return to the ground state, but not necessarily in one leap. It could conceivably drop all the way back releasing a photon of the same frequency and energy as the one that struck the electron boosting it out of the ground state. Or it might drop to the 2nd energy level first (***B***), and then finally drop back to the 1st energy level (***C***). This two-step return will result in two photons that have different frequencies, and these photons are also different from the photon that hit the electron initially. ***Photons are emitted from the atom***. Here are some of the energy levels of a hydrogen atom. The atom is small, so energy is measured in electron volts.

Drawing a circular atom is tedious. The following covers the same scenario as detailed above, but includes the mathematical steps and an additional step ***D***. Remember this is only one scenario, there are many energy levels.



A: ***Absorption*** of a photon. The hydrogen atom is bombarded with light of frequency, $2.92\text{x}10^{15}$ *Hz*.

$$E = hf = \left(4.14\times10^{-15} eV\cdot s\right)\left(2.92\times10^{15} Hz\right) = 12.1 eV$$

As a result an electron in the ground state, -13.6 *eV* is boosted to the second energy level, -1.5 *eV*. In this case it drops to the first energy level and then drops back to the ground state. Lot's of possibilities exist.

B: ***Emission*** of a photon. The electron falls to the first energy level, a drop of 1.9 *eV*. This corresponds to a frequency of,

$$f = E/h = \left(1.9eV\right)\Big/\left(4.14\times10^{-15} eV\cdot s\right) = 4.59\times10^{14} Hz$$

A photon of light is emitted from the atom with this frequency. This ***generates a distinct band of light on the emission spectrum***.

C: ***Emission*** of a photon. Next the electron falls back to the ground state, a drop of 10.2 *eV*. This corresponds to a frequency of,

$$f = E/h = \left(10.2eV\right)\Big/\left(4.14\times10^{-15} eV\cdot s\right) = 2.46\times10^{15} Hz$$

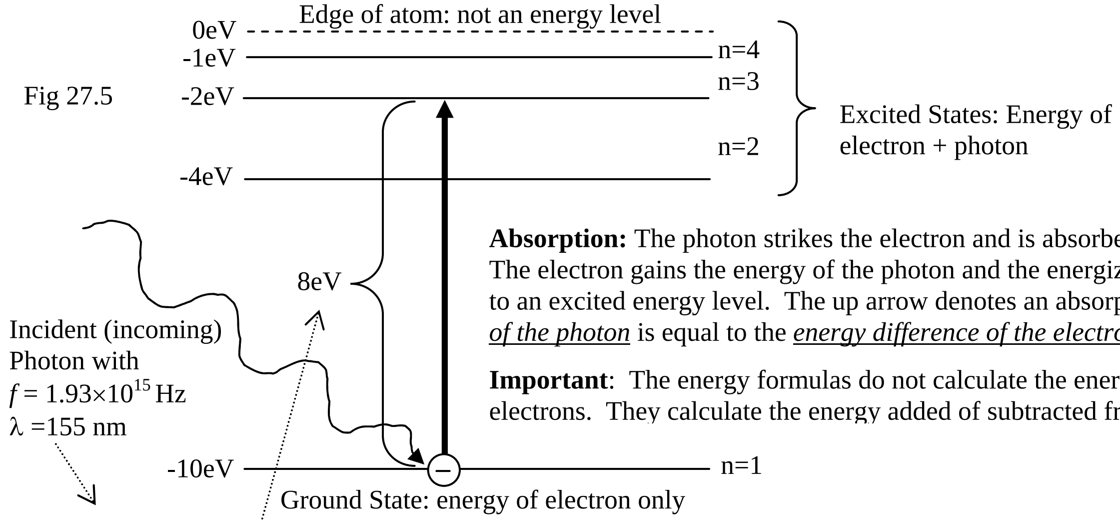
A photon of light is emitted and this ***generates a distinct band of light on the emission spectrum***.

D: ***Ionization***. If a photon strikes a hydrogen atom electron in the ground state with more than 13.6 *eV* the electron receives enough energy to leave the atom entirely. The hydrogen atom lacking its electron becomes an ion. Therefore, this energy is referred to as the ***ionization energy***. (Also called the ***work function***. See below.)

Example 27.1: Energy Levels: Absorptions

This hypothetical atom is struck by a photon, f = 1.93×10^{15}. Fig 27.5 details the flow of energy.

Fig 27.5



Absorption: The photon strikes the electron and is absorbed by the electron. The electron gains the energy of the photon and the energized electron moves to an excited energy level. The up arrow denotes an absorption. The *energy of the photon* is equal to the *energy difference of the electron*.

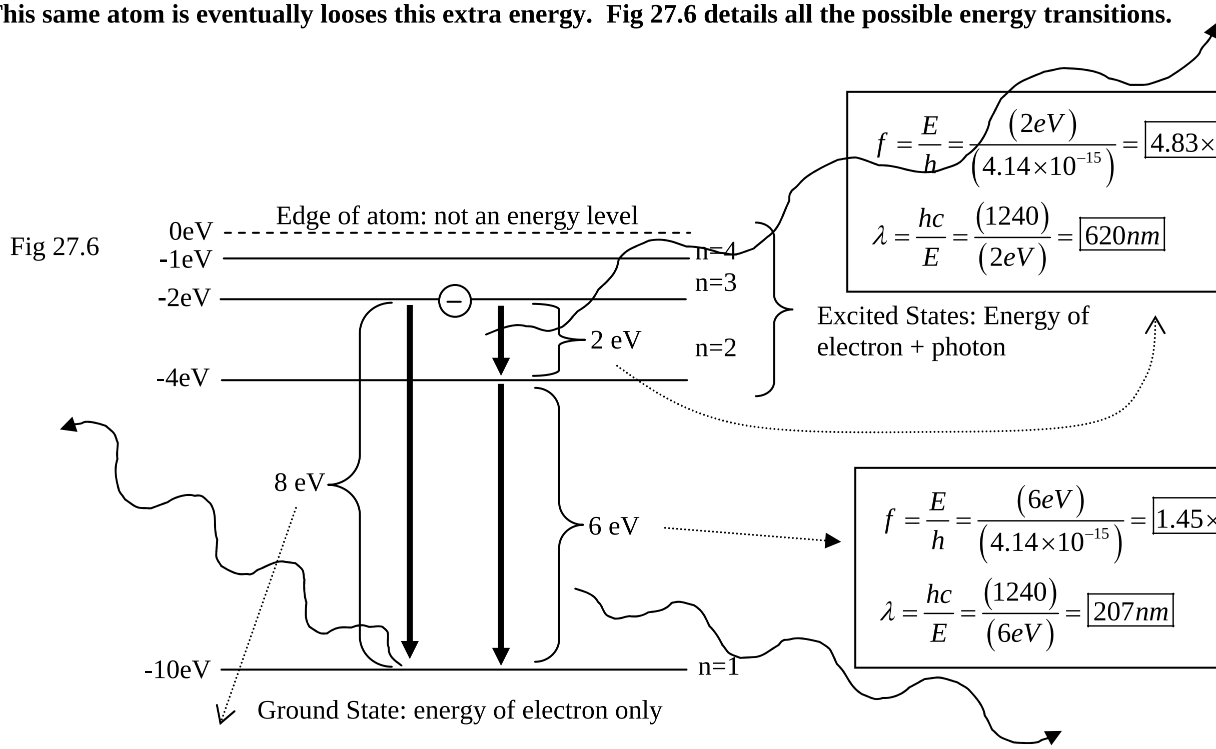
Important: The energy formulas do not calculate the energy levels of electrons. They calculate the energy added of subtracted from electrons.

$$E = hf = \left(4.14\times10^{-15}\right)\left(1.93\times10^{15}\right) = \boxed{8eV} \quad and \quad E = \frac{hc}{\lambda} = \frac{(1240)}{(155)} = \boxed{8eV}$$

Example 27.1: Energy Levels: Emissions

This same atom is eventually looses this extra energy. Fig 27.6 details all the possible energy transitions.

Fig 27.6



$$f = \frac{E}{h} = \frac{(2eV)}{\left(4.14\times10^{-15}\right)} = \boxed{4.83\times10^{14}Hz}$$

$$\lambda = \frac{hc}{E} = \frac{(1240)}{(2eV)} = \boxed{620nm}$$

$$f = \frac{E}{h} = \frac{(6eV)}{\left(4.14\times10^{-15}\right)} = \boxed{1.45\times10^{15}Hz}$$

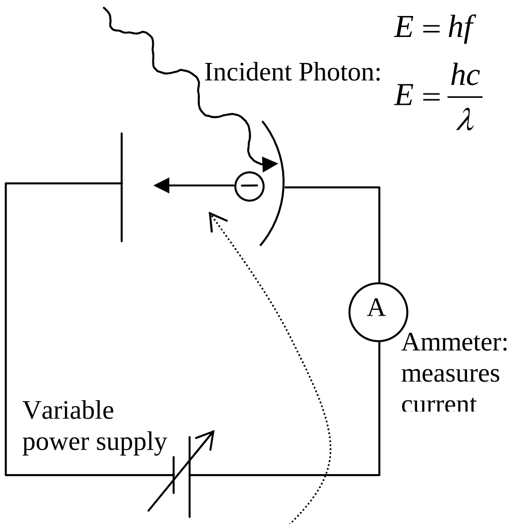
$$\lambda = \frac{hc}{E} = \frac{(1240)}{(6eV)} = \boxed{207nm}$$

$$f = \frac{E}{h} = \frac{(8eV)}{\left(4.14\times10^{-15}\right)} = \boxed{1.93\times10^{15}Hz}$$

$$\lambda = \frac{hc}{E} = \frac{(1240)}{(8eV)} = \boxed{155nm}$$

Emission: The electron drops to a lower energy level, loosing energy in the form of a photon. The electron can drop to any level, or combination of levels, below its current level. This means that one wavelength and frequency may be absorbed while others are emitted. Since some of the drops are shorter than the original absorption they do not involve as much energy. Therefore the emitted photons can have lower frequencies and longer wavelengths.

Photoelectric Effect: Young's Double Slit Diffraction experiment provided evidence that light exhibited wave properties. The photoelectric effect provided evidence of lights particle behavior. If light is shined on certain photoelectric materials a current can be induced. This requires the light to have enough energy to knock electrons out of the atoms, so a current can flow. This is the basis of solar energy. This process requires a certain minimum energy, known as the work function ϕ. So if a photon $\boxed{E = hf}$ strikes a photoelectric material some of its energy is required to move the electron completely out of the atom. This is the energy needed to ionize the atom. The kinetic energy given the electrons is the energy left over after the ionizing energy, work function, is subtracted from the photons energy. $\boxed{K_{\max} = hf - \phi}$



$$\boxed{\begin{aligned} K_{\max} &= E_{incident\ photons} - \phi \\ K_{\max} &= hf - \phi \\ K_{\max} &= \frac{hc}{\lambda} - \phi \end{aligned}}$$

In photoelectric effect a photon hits a photo (light) sensitive substance. If the light hits with sufficient energy it boosts electrons past all the energy levels and the edge of the atom. If there is any extra energy left over after reaching the edge of the atom the electrons keep the excess energy. The excess energy is kinetic energy as the electrons are knocked out of the atom. If electrons have kinetic energy they are moving. If they are moving they are a current. A current flowing means electricity is flowing.

Stopping potential (V_S): The variable power supply is put in the circuit to test the strength of the electric current created by the photon collisions. The power supply is put in backwards and is adjusted until the current in the circuit is zeroed out (ammeter reads zero). If it takes 4.5V volts of electric potential (pressure) to cancel (stop) the photoelectrons from flowing then the photoelectric effect must be creating a potential of 4.5V in the opposite direction.

Kinetic Energy ($K_{\max}$): The amount of kinetic energy the electrons have is easily determined from the stopping potential. If V_S = 4.5V, then $K_{\max}$ = 4.5eV. These are different variables with different units, but conveniently they share the same numerical magnitude (value).

Work Function (ϕ): The amount of energy needed (work that must be done) to get the electron out of the atom. Also known as ionization energy.

$K_{\max} = hf - \phi$: The energy that electrons have when they leave the atom depends on how much energy the photon had that hit it and how much of the photons energy was used to get the electron out of the atom.

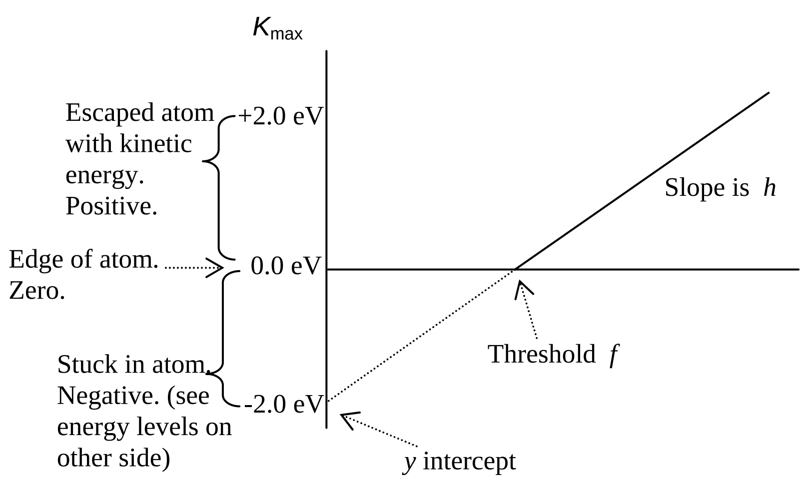
Threshold (minimum or cutoff) **frequency**: If the photon has too little energy the photoelectrons do not get out of the atom. So there is no $K_{\max}$.

$K_{\max} = hf - \phi \qquad\qquad 0 = hf - \phi$

Graphing Photoelectric Effect

$K_{\max} = hf - \phi$ is the equation of a line.

$y = mx + b$



5–28 Nuclear Physics

Nucleons: **Proton** Positive charge $+e = 1.6 \times 10^{-19}\ C$ $m_p = 1.6726 \times 10^{-27}\ kg$
Neutrons Neutral charge $q = 0$ $m_n = 1.6749 \times 10^{-27}\ kg$

???? Mass difference between a proton & a neutron ($1.6749 \times 10^{-27} - 1.6726 \times 10^{-27} = 2.3 \times 10^{-30}$)

It is almost the mass of an electron. A neutron is made of a proton and an electron that have fused. This explains why the neutrons are neutral. As for the mass not quite adding up, see $E = mc^2$.

Atomic Mass Number	(A)	= # Nucleons	$^{A}_{Z}X$	$^{12}_{6}C$	or just ^{12}C
Atomic Number	(Z)	= # Protons			
Neutron Number	($N = A\text{-}Z$)	= # Neutrons			

Isotopes: Same # of Protons, different # of Neutrons $^{12}_{6}C$ $^{13}_{6}C$ $^{14}_{6}C$

Unified Atomic Mass (u)

$C\text{-}12 = 12.00000\ u$ $1.00000\ u = 1.66054 \times 10^{-27}\ kg$

$E = m c^2$ $100000\ u = 931.5\ MeV$

Binding Energy: Total mass of a stable nucleus is less than the component protons and neutrons

He: lists as 4.002602 *u*, but when the protons, neutrons, and electrons are added separately it totals to: 4.032980 *u*

When *He* is formed some mass turns into energy: $\Delta E = \Delta m\, c^2$, ***Mass Defect*** $= \Delta m = 28.30\ MeV$

Total Binding Energy: Mass difference, amount of energy required in order to break the nucleus apart.

Average Binding Energy per Nucleon: Divide Total binding energy by the number of nucleons.

Highest for Iron: Requires the most energy to split this nucleus, most stable

Hydrogen: Used in fusion, Energy drops from ***Fe*** to ***H***. Binding energy released: Powers stars

Uranium: Used in fission, Energy drops from ***Fe*** to ***U***. Binding energy released: Nuclear power & bombs

Strong Nuclear Force: Force holding nucleons together

Radioactivity: Disintegration / decay of unstable nucleus. Certain isotopes are unstable and emit rays (radiation).

- The ***Electric Force*** acts over entire nucleus. It results from the repulsion of all the protons. Neutron are not involved in the electric force.
- The ***Strong Force*** is a short-range force. It holds the individual nucleons together. Both neutrons and protons attract due to this force.
- So adding more neutrons can help hold the nucleus together, since they do not contribute to the repulsive electric force. There is however an ideal amount of neutrons, not too many and not too few that each nucleus needs in order to be stable. If this number is out of balance the nucleus can deteriorate spontaneously.
- The larger the nucleus the greater the electric force. The distances become too large for the strong force to hold the nucleons together. The balance of force favors electric force.
- **Transmutation*:*** Occasionally parts of the nucleus are repelled out with great force and speed. When part of the nucleus is ejected the ***Parent nucleus*** changes into a ***Daughter nucleus***

Alpha (α) Decay: Results when an ***α Particle***, *He* nucleus $\boxed{^{4}_{2}He}$ is spontaneously ejected from the nucleus. The nucleons comprising the alpha particle are strongly bonded to each other and they eject as a single packet.

$$^{226}_{88}Ra \rightarrow {}^{222}_{86}Rn + {}^{4}_{2}He$$

Beta (β) Decay: Results when there are too many neutrons compared to the ideal number required to maintain the electric / strong force balance. A neutron is a proton and an electron that have been fused together. So if there are too many neutrons one of them can split, forming a new proton (adding to atomic number) and a nuclear electron, known as a ***β particle*** $\boxed{^{0}_{-1}e}$. This particle has a very low mass and picks up all the energy of this transmutation. It is ejected at an extreme speed. Being smaller and faster than the alpha particle it has more penetrating power and is thus a more hazardous form of radiation. $^{14}_{6}C \rightarrow {}^{14}_{7}N + {}^{0}_{-1}e + \bar{v}$. One by product of this reaction is a positron (anti-electron). The existence of this anti-matter particle lead scientists to postulate the existence of a fourth fundamental force, the ***Weak Nuclear Force***. Don't worry about the positron on this exam.

Gamma Rays: Electrons around the nucleus can be found in excited states. It turns out that nucleons can exist in excited states as well. If a nucleon drops to a lower energy state a tremendous energy is released, since the energy differences in the nucleus are huge. This results in the emission of extremely high energy photons.

Half Life: Length of time for half of the sample to decay. See worksheet that follows this section.

Law of Conservation of Nucleon Number: Total # of nucleons remains constant

Nuclear Reactions: Nucleus is struck by a particle. Causes transmutation

Enrico Fermi: Neutrons most effective projectile particle. No charge, not repelled.

Fission

Hahn and Strassmann: Bombardment sometimes made smaller particles

Meitner and Frisch: Realized that Uranium split in two

Liquid drop model: Neutron added to nucleus increases energy. Increases motion of individual nucleons. Abnormal elongated shape. Short range Strong Force is weakened. Electric Repulsive Force dominates. 2 Fission Fragments result. Also some free neutrons are given off. Tremendous amount of energy released. Fission fragment plus neutron are substantially lower in energy than original U 235

Chain Reaction: 2 to 3 neutrons freed collide with other ^{235}U

Self Sustaining Chain Reaction: Nuclear Reactor

Moderator: Need slow neutrons, with the right speed

Enriched Uranium: To increase probability of ^{235}U fission

Critical Mass: Mass of fuel large enough to compensate for lost neutrons

Multiplying Factor: 1 or more neutrons must go to next reaction.

Subcritical: Less than one neutron goes on

Supercritical

Control Rods: Cadmium or Boron. Absorb neutrons

Delayed Neutrons: Come from fission fragments

Core: Fuel plus moderator. 2 - 4 % ^{235}U

Problems

Thermal pollution: Disposal of radioactive fission fragments. Radioactive interaction with structural components. Accidental release of radioactivity into atmosphere. Leakage of radioactive waste. Life time of 30 yrs due to build up of radioactivity. Earthquakes. Limited supply of fissionable materials

Breeder Reactor: Some neutrons produced are absorbed by ^{238}U. ^{239}Pu is produced, and is fissionable. So the supply of fuel can increase 100 times. However, Plutonium is highly toxic and can readily be used in bombs, and it involves a graphite moderator, as was used in Chernobyl.

Fusion: Building up nuclei

Sum of energy of nucleus is less than sum of energy of its component parts

Elements may be result of fusion in stars

Proton Proton Cycle

$${}_1^1H + {}_1^1H \rightarrow {}_1^2H + e^+ + \nu \qquad \textbf{0.42 } \boldsymbol{MeV}$$

$${}_1^1H + {}_1^2H \rightarrow {}_2^3He + \gamma \qquad \textbf{5.49 } \boldsymbol{MeV}$$

$${}_2^3He + {}_2^3He \rightarrow {}_2^4He + {}_1^1H + {}_1^1H \qquad \textbf{12.86 } \boldsymbol{MeV}$$

$$4\,{}_1^1H \rightarrow {}_2^4He + 2e^+ + 2\nu + 2\gamma$$

Carbon Cycle: Similar method in hotter stars, can make heavier elements

Thermonuclear Device: Fusion Devices

- Stars are under tremendous gravity
- Creates tremendous pressure
- High pressure means high temperature
- High temperature means particles collide violently

On earth high temperatures and densities not easily achieved

Fission Bomb can ignite Fusion Bomb: Thermonuclear Device or H bomb

Plasma: Need high density as well as temperature. Ordinary materials cannot contain plasma

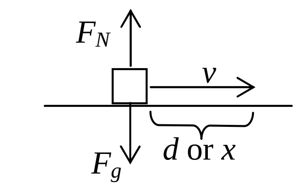
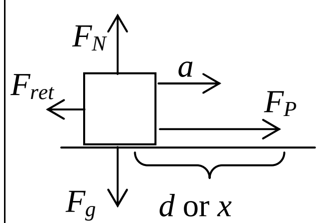
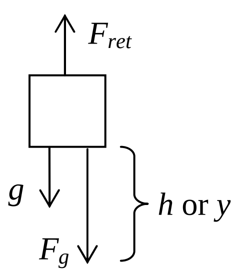
Magnetic Confinement: Tokamak Reactor. **Break even point:** Output equals input

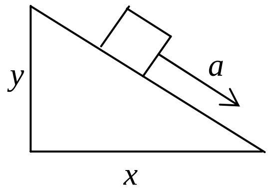
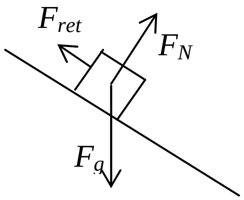
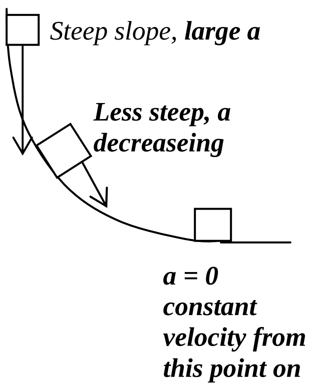
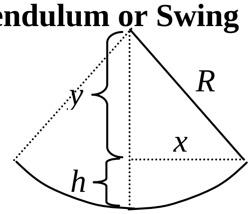
Inertial Confinement: Pellet of deuterium and tritium. Struck simultaneously by several lasers

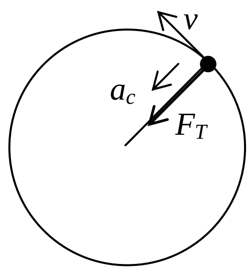
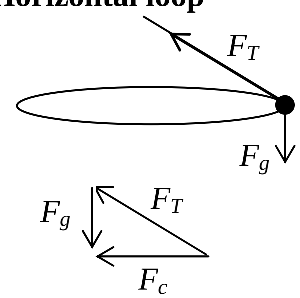
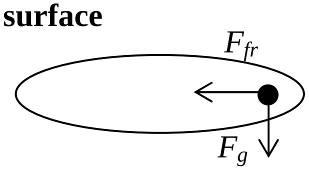
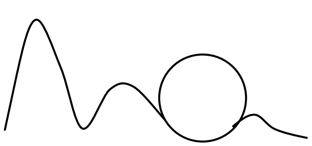
Unit 6 Conclusion

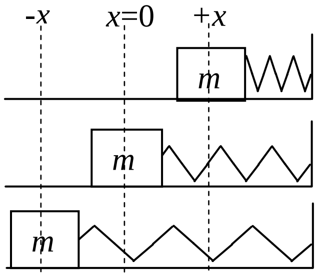
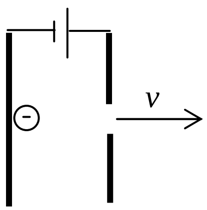
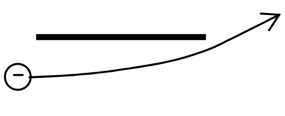
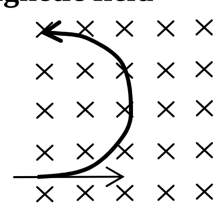
6–29 Kinematics, Force, and Energy Comparison

What is the object doing? What direction is it moving in (if two find *x* and *y* components)? Is it moving at constant *v* (this includes $v = 0$)? Is it accelerating? **What is causing it to do that?** Force? Energy change? See if energy solves the problem first. Then think force and kinematics. Some common motions are discussed in the following pages.

Situation	Kinematics	Force	Energy
Constant Velocity  	Need constant velocity $\boxed{v = \frac{x}{t}}$ or $x = vt$	$\boxed{\sum F = 0}$. Forces are vertical, while motion is horizontal $\boxed{F_g = mg}$ and $\boxed{F_N = mg}$	Inertia only. No force. No energy needed. $W = F \cdot d\cos\theta$ Where θ is the angle between F and d vectors. $W = F \cdot d\cos 90^o$ $\boxed{W = 0}$
Accelerating in *x*  	$\boxed{v_x = v_{ox} + at}$ $\boxed{{v_x}^2 = {v_{ox}}^2 + 2a\left(x - x_o\right)}$ $\boxed{x = x_o + v_{ox}t + \frac{1}{2}at^2}$ In projectile motion $\boxed{x = v_{ox}t}$	There is a sum of force $\boxed{\sum F = ma}$ $\sum F_x = F_{p_x} - F_{retarding_x}$ example $\sum F_x = F_{p_x} - F_{fr}$	A force through a distance. $W_x = \sum F_x \cdot d\cos\theta$ Where θ is the angle between F and d vectors. $W = F \cdot d\cos 0^o$ No retarding forces present $\boxed{W = F \cdot d}$ Now work is done, so there is a Δ energy. $W = \Delta Energy$ $\boxed{W = \Delta K = \frac{1}{2}m\left(\Delta v\right)^2}$ The only thing changing is velocity, so *K* is changing.
Accelerating in *y*  	$\boxed{v_y = v_{oy} + gt}$ $\boxed{{v_y}^2 = {v_{oy}}^2 + 2g\left(y - y_o\right)}$ $\boxed{y = y_o + v_{oy}t + \frac{1}{2}gt^2}$ Horizontal projectile or dropped object $\boxed{y = \frac{1}{2}gt^2}$	$\sum F_y = F_g - F_{retarding_y}$ example $\sum F_y = F_g - F_{air\ resistance}$	$W_y = \sum F_y \cdot h$ Includes retarding forces $W = F_g \cdot h$ No retarding forces $\boxed{W = \Delta U_g = mg\Delta h}$ Height is changing, so *U* is changing. $\boxed{W = \Delta K = \frac{1}{2}m\left(\Delta v\right)^2}$ Velocity is changing, so *K* is changing. $mg\Delta h = \frac{1}{2}m\left(\Delta v\right)^2$ $\boxed{mgh_i + \frac{1}{2}mv^2{}_i = mgh_f + \frac{1}{2}mv^2{}_f}$

Situation	Kinematics	Force	Energy
Inclines  	Kinematic Equations Apply $v_x = v_{ox} + at$ $v_x^{\ 2} = v_{ox}^{\ 2} + 2a\left(x - x_o\right)$ $x = x_o + v_{ox}t + \frac{1}{2}at^2$ $v_y = v_{oy} + gt$ $v_y^{\ 2} = v_{oy}^{\ 2} + 2g\left(y - y_o\right)$ $y = y_o + v_{oy}t + \frac{1}{2}gt^2$	Motion is parallel to slope Acceleration down the slope is caused by the addition of F_g and F_N. The resultant of these two vectors is $F_g sin\theta$. Since the natural motion is down the slope set that direction as +. In some problems it is useful to reverse this (if the object is going up hill). $\boxed{\sum F_{\parallel} = F_g \sin\theta - F_{retarding}}$ $F_N = mg\cos\theta$	Work and energy can work parallel, in the *x*, and in the *y* $W = F_{\parallel} \cdot d_{\parallel}$ Force and distance vectors form similar triangles. $W = F_x \cdot d_x$ Work depends on *F* and *d* being parallel. $W = F_y \cdot d_y$ So any pair of parallel vector will solve the problem $\boxed{W = \Delta K = \frac{1}{2}m\left(\Delta v\right)^2}$ $\boxed{W = \Delta U_g = mg\Delta h}$ $mg\Delta h = \frac{1}{2}m\left(\Delta v\right)^2$ $\boxed{mgh_i + \frac{1}{2}mv^2{}_i = mgh_f + \frac{1}{2}mv^2{}_f}$
Down a curve 	Kinematics Fail The net force is changing as the vectors F_g and F_N change. In addition the direction is changing. Acceleration is changing. The Kinematic Equations are designed for changing velocity, but only work for uniform (constant) acceleration.	Force Fails The net force is changing as the vectors F_g and F_N change.	Energy is directionless. $\boxed{W = \Delta K = \frac{1}{2}m\left(\Delta v\right)^2}$ $\boxed{W = \Delta U_g = mg\Delta h}$ $mg\Delta h = \frac{1}{2}m\left(\Delta v\right)^2$ $\boxed{mgh_i + \frac{1}{2}mv^2{}_i = mgh_f + \frac{1}{2}mv^2{}_f}$ A very important case. $\boxed{mgh_{top} = \frac{1}{2}mv^2{}_{bottom}}$ No initial velocity moving to a height of zero.
Pendulum or Swing  $h = R - y$ $y = \sqrt{R^2 - x^2} = R\cos\theta$	Kinematics Fail See accelerating down a curve above. The velocity is zero at either end. The velocity is greatest at the lowest point	Force Fails See accelerating down a curve above. The restoring force is greatest at the ends, as is the acceleration. The restoring force is zero in the middle, and so is the acceleration.	Energy is directionless. $\boxed{W = \Delta K = \frac{1}{2}m\left(\Delta v\right)^2}$ $\boxed{W = \Delta U_g = mg\Delta h}$ $\boxed{mgh_{top} = \frac{1}{2}mv^2{}_{bottom}}$

Situation	Kinematics	Force	Energy
Object on string, Vertical loop 	Tangential Velocity Instantaneous velocity is tangent to the circular motion. $\boxed{v = \frac{2\pi r}{T}}$ Acceleration is toward center, centripetal. $\boxed{a_c = \frac{v^2}{r}}$	Center seeking. Force is centripetal, F_c, is the sum of force in circular motion. Toward center is +. Find tension at the top. $\boxed{F_c = F_T + F_g}$ Find tension at the bottom. $\boxed{F_c = F_T - F_g}$	Unlike previous scenarios, the object definitely has velocity at the top. There is a height difference from top to bottom, but the object has speed at the top as well. And the bottom may not necessarily be the lowest point in the problem $\boxed{mgh_{top_i} + \frac{1}{2}mv_{top}{}^2{}_i = mgh_{bottom\,f} + \frac{1}{2}mv_{bottom}{}^2{}_f}$
Object on string, Horizontal loop 	Tangential Velocity Instantaneous velocity is tangent to the circular motion. $\boxed{v = \frac{2\pi r}{T}}$ Acceleration is toward center, centripetal. $\boxed{a_c = \frac{v^2}{r}}$	Center seeking. Force is centripetal, F_c, is the sum of force in circular motion. Toward center is +. Find F_c by adding vectors (tip to tail). Then solve for F_T. $\boxed{F_T = \sqrt{F_c{}^2 + F_g{}^2}}$	No work is done $W = F \cdot d\cos\theta$ Where θ is the angle between F and d vectors. $W = F \cdot d\cos 90^o$ $\boxed{W = 0}$ At any instant the direction of motion (tangent to the circle) is perpendicular to the center seeking F_c, the F_T, and the F_g. And for one revolution there is no total displacement from the origin, since a single revolution brings you back to the starting point.
Object turning on flat surface 	Tangential Velocity $\boxed{v = \frac{2\pi r}{T}}$ $\boxed{a_c = \frac{v^2}{r}}$	Why is there circular motion? Object not sliding off disk, or car turning on a road. $\boxed{F_c = F_{fr}}$	No work is done See above.
Roller Coaster 	Need uniform slope Kinematics only work on sections that have constant slope. If the track is curved try energy.	Force centripetal in the loop. To find the speed needed to have passengers feel weightless at the top of the loop $\boxed{F_c = F_g}$	Energy works everywhere, with its directionless advantage. You can solve for any point A using any other point B. Use the complete equation. The car will usually have both speed and height at every point. An exception is the lowest point on the track, or if it starts with zero velocity at the top of a hill (this is unlikely since roller-coasters don't stop at the top of each hill). If it has velocity and height at the top you need to include both. $\boxed{mgh_A + \frac{1}{2}mv_A{}^2 = mgh_B + \frac{1}{2}mv_B{}^2}$

Situation	Kinematics	Force	Energy
Spring 	Kinematics Fail See Down a curve, and Pendulum above. The velocity is zero at maximum +/- *x* (amplitude) The velocity is greatest at *x* = 0	Force Fails See Down a curve, and Pendulum above. The restoring force is greatest at maximum +/- *x* (amplitude). The restoring force is zero at *x* = 0, and so is the acceleration.	Energy is directionless. $\boxed{W = \Delta K = \frac{1}{2}m(\Delta v)^2}$ $\boxed{W = \Delta U_S = \frac{1}{2}k(\Delta x)^2}$ If it starts at maximum *x* (amplitude) and it converts all the springs energy into speed of the object pushed / pulled by the spring $\boxed{\frac{1}{2}kx^2 = \frac{1}{2}mv^2}$ $\frac{1}{2}kx^2{}_i + \frac{1}{2}mv^2{}_i = \frac{1}{2}kx^2{}_f + \frac{1}{2}mv^2{}_f$
Particle accelerated by electric field 	Potential difference. Velocity increases. Positive charges go in the opposite direction. But, + particles are more massive, don't accelerate as quickly, and have lower final velocities.	The force of the electric field $\boxed{F = qE}$	Electromagnetism: New forms of energy, but energy is still conserved. $W = \Delta Energy$ $\boxed{W = \Delta K = \frac{1}{2}mv^2}$ $\boxed{W = \Delta U_E = qV}$ $\boxed{qV = \frac{1}{2}mv^2}$ $qV_i + \frac{1}{2}mv^2{}_i = qV_f + \frac{1}{2}mv^2{}_f$
Charged particle parallel to plates 	It acts like a projectile $\boxed{x = v_{xo}t}$ $\boxed{y = \frac{1}{2}at^2}$ get *a* from *F*	Electric field is perpendicular Particle is forced toward the plate with opposite sign. $E = \frac{F}{q}$ $\boxed{F = qE}$ $\boxed{\sum F = ma}$ $ma = qE$	Work is done in the direction of the electric field. $W = F \cdot d$ $W = qE \cdot d$
Charged particle in a magnetic field 	Path curved by field. If the field is large enough the particle will follow a circular path. $\boxed{v = \frac{2\pi r}{T}}$ $\boxed{a_c = \frac{v^2}{r}}$	Forced to center by field. $\boxed{F_c = F_B}$ $\boxed{m\frac{v^2}{r} = qvB}$	No work is done $W = F \cdot d \cos 90^o$ $\boxed{W = 0}$ At any instant the direction of motion (tangent to the circle) is perpendicular to the center seeking F_c, and the F_B.